3-D Magnetic Field Calculation in Aerial Toroidal Coil System Arrangement

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ABSTRACT

The aim of proposed paper is to present the magnetic field calculation in aerial toroidal and poloidal coil systems arrangement. The magnetic system is in principal very similar to the Tokamak configuration. The coil models have been done in 2-D axisymmetry [1] and/or in 3-D [2], depending on geometric arrangement and excitation of coils. The finite element method (FEM) was used in all analyses. The calculation of 2-D and 3-D electromagnetic quantities were performed in magneto-static mode and also in magneto-transient mode coupled with external circuit. Poloidal coil arrangement and magnetic field calculation were performed in 2-D axisymmetry in magneto-static mode. Meanwhile, the toroidal coil system requires a 3-D geometric model description.

1  INTRODUCTION

The coil system analyzed in our study is composed of toroidal and poloidal coils. Principal design example is demonstrated on Fig. 1. Different geometric arrangements were used to calculate and study the magnetic field distribution. The operation of poloidal coil arrangement was simulated by finite element method in:

- 2-D axisymmetry in magneto-static mode with coil arrangement and excitation current values taken from [3],
- 2-D axisymmetry in magneto-static mode (the same system arrangement as previously) with constant current excitation in central region (plasma region), meanwhile, the toroidal coil arrangement was simulated in:
- 3-D magneto-transient mode coupled with external circuit.

The system composed of poloidal and toroidal coil arrangement with current in central region (plasma region) was also studied. The computation of such complex system was performed in 3-D magneto-static mode for different values of poloidal coils currents with toroidal coils current and central region current kept constant. It has to be pointed out, that the medium of all analyses was vacuum and applied material for all coils was set to ideal conductor.

Figure 1: Basic scheme of Tokamak and coil system arrangement [4].

2 2-D AXISYMMETRY POLOIDAL COIL ARRANGEMENT

The poloidal coil arrangement in a coil system presented in Fig. 1 allows the use of axisymmetry. The coils are wound around central axis and current in central region is of the same geometry type. All poloidal coils are geometrically placed as in [3] and supplied with constant current as shown in Table 1. Previously defined model was calculated in magneto-static mode, this means that all the electric and magnetic quantities are constant in time. The theoretical equation which governs this mode of calculation is:

\[
\text{curl} \left( \frac{1}{\mu} \text{curl} (A) \right) = \vec{J} - \text{curl} (H_C),
\]

where the variable \( \vec{A} \) is magnetic vector potential, \( \vec{J} \) current density normal to the plane and \( H_C \) coercive field of a magnet, if there is any in a magnetic system.
Table 1: Geometry of poloidal coil arrangement and current values.

<table>
<thead>
<tr>
<th>Coil No.</th>
<th>R [m]</th>
<th>Z [m]</th>
<th>I [MA]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.25</td>
<td>0.25</td>
<td>-0.62</td>
</tr>
<tr>
<td>2</td>
<td>2.25</td>
<td>0.75</td>
<td>-1.053</td>
</tr>
<tr>
<td>3</td>
<td>2.25</td>
<td>1.25</td>
<td>-1.513</td>
</tr>
<tr>
<td>4</td>
<td>2.25</td>
<td>1.75</td>
<td>-0.665</td>
</tr>
<tr>
<td>5</td>
<td>2.25</td>
<td>2.25</td>
<td>-0.665</td>
</tr>
<tr>
<td>6</td>
<td>2.25</td>
<td>2.75</td>
<td>1.184</td>
</tr>
<tr>
<td>7</td>
<td>2.25</td>
<td>3.25</td>
<td>3.36</td>
</tr>
<tr>
<td>8</td>
<td>3.25</td>
<td>5.75</td>
<td>6.348</td>
</tr>
<tr>
<td>9</td>
<td>3.75</td>
<td>6.3</td>
<td>6.518</td>
</tr>
<tr>
<td>10</td>
<td>5.25</td>
<td>6.3</td>
<td>4.81</td>
</tr>
<tr>
<td>11</td>
<td>5.75</td>
<td>6.25</td>
<td>3.643</td>
</tr>
<tr>
<td>12</td>
<td>7.5</td>
<td>5.65</td>
<td>-3.276</td>
</tr>
<tr>
<td>13</td>
<td>8</td>
<td>5.4</td>
<td>-5.877</td>
</tr>
<tr>
<td>14</td>
<td>8.5</td>
<td>5.1</td>
<td>-8.624</td>
</tr>
</tbody>
</table>

In Fig. 2 are shown the flux lines (vector of magnetic flux density is tangential to the flux line in every point) for the currents in poloidal coils as described in Table 1. The geometry (Fig. 2) is presented in full 2-D axisymmetry view. Fig. 3 shows the distribution of magnetic flux density in form of vectors for upper quarter of the model. As we can see, there is a quite uniform magnetic field distribution in axisymmetrical central region. In this central region, the toroidal coils (not in this analyse) are placed to shape the toroidal magnetic field. Also, this is the region where the plasma takes place. In this region we apply central poloidal current (15 MA) to analyse the interaction between magnetic field produced by poloidal coils and magnetic field produced by central poloidal current. The results are shown in Fig. 4. The Fig. 5 presents the magnetic flux lines produced by current excited poloidal coils and by central poloidal current in full 2-D axisymmetry view. There can be observed well known X-points.

Figure 2: The flux lines for the current excited poloidal coils as described in Table 1.
Figure 3: Magnetic flux density distribution in poloidal coils surrounding.

Figure 4: Interaction between magnetic flux density produced by poloidal coils and magnetic flux density produced by central poloidal current.
3-D TOROIDAL COIL ARRANGEMENT

3-D aerial toroidal coil system arrangement is presented in Fig. 6. It consists of eighteen (nine of them are shown) equidistantly placed excitation coils. The coils are supplied with constant current. Due to the nature of excitation currents the 3-D magnetic analysis was magneto static. The results presented in Fig. 6 show magnetic flux density distribution in central region of toroidal coil systems arrangement. Meanwhile, the lines in Fig. 7 show the magnetic flux distribution in surroundings of coil system. In Fig. 6 and Fig. 7 can be clearly seen the non-homogeneous magnetic flux density distribution in central toroidal region. The scale on the left side of Fig. 6 and Fig. 7 shows the values of magnetic flux density between 2 T and 3.5 T.
3.1 3-D toroidal coil arrangement coupled with external circuit

The magneto-transient module allows the study of electromagnetic structure in transient state. The current sources can vary with time, and their values can be described by a formula. This module is particularly useful with circuit coupling or with kinematics coupling, or with these two couplings, as soon as eddy current develops in the conducting regions of electromagnetic structure. These couplings allow us to study presented aerial coils system arrangement in a transient state. The theoretical equation which is used in this mode of calculation is:

$$\sigma \frac{d\vec{A}}{dt} + \text{curl} \left( \frac{1}{\mu} \text{curl}(\vec{A}) \right) = \vec{J} - \text{curl}(\vec{H}_c),$$

where $\sigma$ is electric conductivity, the variable $\vec{A}$ is magnetic vector potential, $\vec{J}$ current density normal to the plane and $\vec{H}_c$ coercive field of a magnet.

The quantities that can be calculated in transient-magnetics are magnetic flux density, time dependent supply current, time dependent eddy currents distribution in conducting region and Joule losses, flux across a surface or in a coil, mutual inductances, energies, forces, ...

These quantities can be presented in the form of time dependent curves or surfaces. In our case, the external circuit was composed of capacitor, diode and a switch as shown in Fig. 8. At the start of simulation the switch was turned on and the current flow through the external circuit and through eighteen coils connected in series. In this calculation the toroidal coils were made from copper. Time depending line current in 3-D toroidal coil arrangement coupled with external circuit is shown in Fig. 9. Current value and its shape depend on resistance and inductance of coil system, capacitor voltage and capacitance.

Figure 8: External circuit coupled with 3-D toroidal coil arrangement.
Time depending current excites also time depending magnetic flux density in presented 3-D toroidal coil arrangement. Fig 10 presents the magnetic flux density distribution at time $t = 0.004s$, meanwhile Fig. 11 shows it at $t = 0.0112s$ and Fig. 12 at $t = 0.0188s$ (maximum current value).

![Figure 9: Time depending line current in 3-D toroidal coil arrangement coupled with external circuit.](image)

![Figure 10: Transient magnetic flux density distribution at $t = 0.004s$.](image)

![Figure 11: Transient magnetic flux density distribution at $t = 0.0112s$.](image)
3.2 3-D toroidal and poloidal coils arrangement with central poloidal current

The study of magnetic field distribution with different current excitation was further studied in 3-D magneto-static mode. The analyses were done in three steps. Firstly, the excitation current was applied just to toroidal coils \((I_t=7 \text{ MA})\) and separately just to the poloidal coils \((I_{p1}=6.384 \text{ MA}, I_{p2}=3.643 \text{ MA}, I_{p3}=-8.624 \text{ MA})\). The magnetic flux distribution as result is shown on Fig. 13 and on Fig. 14. The area with highest values of magnetic flux density is insight the torus shifted between center and inner toroidal radius, this is due to toroidal coil design. It is possible to design the coil which will meet any desired magnetic flux density distribution.
Secondly, the excitation was applied to poloidal coils ($I_{p1}=6.384$ MA, $I_{p2}=3.643$ MA, $I_{p3}=-8.624$ MA) and also the current was assigned to the central (plasma region) torus region ($I_{pl}=15$ MA). The magnetic flux distribution (Fig. 15) on defined cross sections shows the interaction between magnetic fields. Further analyse could be done in a dynamic sense, this means to calculate the forces acting on central torus current (plasma) while the poloidal current is regulated in way to produce maximum position stability of plasma.
And finally, the current excitation was assigned to toroidal coils \( (I_t=7 \text{ MA}) \), poloidal coils \( (I_{p1}=6.384 \text{ MA}, I_{p2}=3.643 \text{ MA}, I_{p3}=-8.624 \text{ MA}) \) and to the central (plasma region) torus region \( (I_{Pl}=15 \text{ MA}) \). The results in form of magnetic flux density are presented in Fig. 16.

![Figure 16: The magnetic flux density distribution due to excitation of toroidal coils, poloidal coils and central torus current excitation.](image)

4. CONCLUSIONS

All the analyses were done in sense to present the happenings in surroundings of different geometric forms currying electric current. The analyses were magneto-static (time independent) and magneto-transient (time dependent). Materials used in some analyses were all ideal (vacuum, ideal conductor), but the materials can be also described with their real characteristics and further analysed. There is also no iron core, but it can be included with all parasitic effects regarding time depending magnetic fields (magnetic saturation, eddy currents, hysteresis losses, …). The coil conductors can be also simulated as they are made from superconducting material. All described parametrically defined geometric forms can be incorporated in different optimization techniques to achieve desired shape of magnetic field or for some other previously defined goals. Coupling between geometric model with external circuit allows us to study the transient effects in materials and also to integrate whole model into modern regulation of applied electric quantities.

REFERENCES