Component unavailability uncertainty and the safety systems unavailability

Andrija Volkanovski
Jožef Stefan Institute
Jamova 39
1000, Ljubljana, Slovenia
Andrija.volkanovski@ijs.si

Blaže Gjorgiev, Duško Kančev
Jožef Stefan Institute
Jamova 39
1000, Ljubljana, Slovenia
Ljubo.Fabjan@ijs.si

ABSTRACT

Fault tree analysis is deductive approach used in the Probabilistic safety analysis (PSA) for assessment of system unavailability. The basic events are the ultimate parts of the fault tree, representing the failures of components or undesired events. The increased and extended application of the PSA requires appropriate consideration of uncertainties in analyses and interpretation of the results. Inadequate treatment of uncertainties may lead to poorly supported or even wrong conclusions whose final consequence is a loss of adequate level of safety.

Epistemic uncertainty results from the imperfect knowledge or incomplete information regarding values of parameters of the underlying model. It is also called state-of-knowledge uncertainty. Epistemic uncertainty is considered in the models by probability distributions associated with uncertain parameters. Probability distributions associated with uncertain parameters represent the state of knowledge about the right values of the parameters and are therefore very often derived from expert judgment.

This paper presents the results of the analysis of the introduction of probability distributions associated with component unavailability parameters, on the overall unavailability of the analyzed system. The normal and lognormal distributions are introduced as probability distributions associated with component unavailability. The auxiliary feedwater system of nuclear power plant is selected as test system for assessment of the uncertainty propagation.

Six case scenarios are developed for assessment of the implications on introduction of different probability distributions for different number and sets of components. Obtained results show that the probability density function of the top event depends on type and parameters of uncertainty distributions as well as importance of the basic events with considered uncertainty. Introduction of lognormal distribution for uncertainty characterization of basic events can result in heavy tails of probability density function and increased likelihood of having top event probability larger than the mean value. The current approaches for consideration of the uncertainties in risk-informed decision making are discussed and the need for their appropriate consideration in risk-acceptance guidelines is emphasized.
1 INTRODUCTION

Appropriate consideration of uncertainties shall be given in probabilistic safety analyses and interpretation of their results. Inadequate treatment of uncertainties may lead to poorly supported or even wrong conclusions whose final consequence is a loss of adequate level of safety. Maintenance of the adequate level of safety is especially important for the nuclear technology as high-consequence technology.

There are two aspects to uncertainty that must be distinguished and treated differently when creating models in probabilistic safety analyses. They are termed aleatory and epistemic uncertainty [1-3]. Aleatory uncertainty results from the effect of inherent randomness or unpredictable variability of the modeled phenomenon. It represents the in deterministic and unpredictable random performance of the system. Epistemic uncertainty results from the imperfect knowledge or incomplete information regarding values of parameters of the underlying model. It is also called state-of-knowledge uncertainty.

Epistemic uncertainty is typically classified into three different classes:

- Parameter uncertainty, associated with imperfect knowledge about the input parameter values used in the analysis.
- Model uncertainty, which exists when there is no consensus approach to modeling specific phenomena or events.
- Completeness uncertainty representing uncertainties due to the portion of risk that is not explicitly included in the analysis.

Parameter uncertainty relates to the uncertainty in the computation of the input parameter values used to quantify the probabilities of the basic events (BE) in the PSA. The parameters uncertainties result from their interdependence with modelling assumptions, lack of statistically significant data, expert opinion and rarity of modelled events [4]. The most of the events in risk models in the PSA are relatively rare resulting with scarce data and significant uncertainties.

The normal and lognormal distribution are two standard distributions used in PSA for consideration of the parameters uncertainties [4,5]. The main characteristics of both distributions are given in following section. Implications on introduction of these probability distributions for different number and sets of components are investigated for reference nuclear power plant safety system model.

2 DISTRIBUTIONS CHARACTERISTICS

2.1 Normal distribution

Normal distribution is statistical distribution identified to be used in standard PSA as distribution of input parameters of the basic events [4,5].

The probability density function of the normal distribution is:

\[ f(x) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left( -\frac{(x - \mu)^2}{2\sigma^2} \right) \]  

where \( x = \) random variable; \( \sigma = \) standard deviation; and \( \mu = \) mean of the distribution.

The main problem with application of normal distribution in PSA is that tail of the distribution extends out in negative domain and this extension can be considerable for large standard deviations [4].
Several statistics measure the goodness-of-fit of the normal distribution to the data. One is a measure of skewness. For a sample of \( n \) values the sample skewness is obtained as:

\[
\text{Skewness} = \frac{\sum (x - \bar{x})^3 / n}{\left( \sum (x - \bar{x})^2 / n \right)^{3/2}}
\]  

(2)

where \( \bar{x} \) = sample mean; \( n \) = sample size.

The skewness indicates departures from symmetry with value of zero for a normal distribution and value near zero for any symmetric data [6]. Negative values indicates distribution skewed left while positive values indicate a distribution skewed right.

The second goodness-of-fit statistic kurtosis is a measure of the packedness of the distribution:

\[
\text{Kurtosis} = \frac{\sum (x - \bar{x})^4 / n}{\left( \sum (x - \bar{x})^2 / n \right)^2}
\]  

(3)

The normal distribution has a kurtosis of 3. A distribution with kurtosis larger than 3, known as positive kurtosis, has heavier tails and a higher peak than the normal, whereas a distribution with kurtosis smaller than 3 has lighter tails and is flatter compared to normal distribution [6].

### 2.2 Lognormal distribution

Lognormal distribution is one of the first distributions used when the data have a distribution skewed to the right [5].

Distribution of failure rates for faults in software systems and semiconductor electronic systems tends to be lognormal [7-9]. This is supported by failure rate distributions observed in experiments and field failure data.

The failure of a certain metal under stress and temperature due to fatigue cracks follow a lognormal distribution [10,11] including the actual crack lengths of the tubes in the steam generators of the pressurized water reactors [12].

If \( X \) has normal distribution with mean \( \mu_x \) and variance \( \sigma_x^2 \) then \( Y \) defined as \( Y = e^X \) has a lognormal distribution [5]:

\[
f(y) = \frac{1}{y \sqrt{2\pi \sigma_x}} \exp \left[ - \frac{(\ln y - \mu_x)^2}{2\sigma_x^2} \right]
\]  

(4)

\( y > 0, -\infty < \mu_x < \infty, \sigma_x > 0 \)

where \( y \) = lognormal distributed variable.

The components reliability data is normally expressed [13] by mean of lognormal distribution \( \mu_y \) and error factor ER. The ER is defined as ration of the 95\% and the 5\% value of variable, thus representing 90\% confidence levels:

\[
ER \equiv \frac{y_{95}}{y_5} = \exp[1.6449\sigma_x]
\]  

(5)

where \( y_{95} = 0.95^{th} \) quantile of lognormal distribution, \( y_{50} = 0.5^{th} \) quantile of lognormal distribution.

The mean value of the normal distribution \( \mu_x \), for given mean value of lognormal distribution \( \mu_y \) and standard deviation \( \sigma_x \) can be obtained as [5]:
The mode is the point of global maximum of the probability density function and for lognormal distribution is given as:

\[
\text{MODE} = \exp(\mu_x - \frac{\sigma_x^2}{2})
\]

The median is defined as 50th percentile of variable and for lognormal distributed variable is given as:

\[
\text{MEDIAN} = \exp(\mu_x)
\]

The mode is smaller than median that is smaller than mean and these differences become more significant with the increase of the standard deviations [14].

2.3 Monte Carlo propagation

Uncertainty propagation can be analyzed by a computerized Monte Carlo technique [14]. The component unavailabilities are sampled from probability distributions and then propagated toward top event where a point value is calculated for system unavailability. The Monte Carlo sampling is repeated a large number of times, and the resultant point values are then used to evaluate the uncertainty of the system unavailability.

Computer code based on Monte Carlo approach was created for the purposes of the study. The top event probability of the analyzed fault tree is obtained with application of the rare event approximation [5].

The inputs in the developed code are the identified minimal cut sets of the analyzed fault tree with main parameters of the basic events including their uncertainty distributions. Program results include main parameters characterizing the unavailability uncertainty of the system including probability density and cumulative density functions of the top event probability.

3 TEST SYSTEM

The auxiliary feedwater system (AFW) of nuclear power plant [15] was selected as test system for assessment of the uncertainty propagation. The analyzed AFW system, as shown on Figure 1, has three trains, two with electric motor driven pumps and one with steam driven pump. All connections to the AFW system of the second unit at the site [15] were removed from the model. In total 1946 minimal cut sets were identified for the fault tree of the AFW system. The analysis was done with the commercial software.

Case scenarios were developed with selection of different distributions for the basic events in the fault tree. The first case scenario Nor_Nor has normal distribution for two basic events with parameters given in Table 1

<table>
<thead>
<tr>
<th>No.</th>
<th>Name</th>
<th>Mean (\mu)</th>
<th>Deviation (\sigma)</th>
<th>Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>CCF-LK-STMBD</td>
<td>1,00E-04</td>
<td>2,00E-05</td>
<td>Normal</td>
</tr>
<tr>
<td>2</td>
<td>CCF-FS-FW3AB</td>
<td>3,50E-04</td>
<td>5,00E-05</td>
<td>Normal</td>
</tr>
</tbody>
</table>
The second case scenario Log_Log has lognormal distribution for two basic events with parameters given in Table 2. The mean value given in Table 2 is the mean of the lognormal distribution of the parameters.

Table 2: Parameters of probability distributions in case scenario Log_Log

<table>
<thead>
<tr>
<th>No.</th>
<th>Name</th>
<th>Mean μ</th>
<th>ErrorF EF</th>
<th>Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>CCF-LK-STMBD</td>
<td>1,00E-04</td>
<td>3,00E+00</td>
<td>LogNorm</td>
</tr>
<tr>
<td>2</td>
<td>ACT-FA-PMP3A</td>
<td>6,00E-04</td>
<td>3,00E+00</td>
<td>LogNorm</td>
</tr>
</tbody>
</table>

Third case scenario Nor_Log combines both types of distributions with parameters given in Table 3. Third column contains standard deviation or error factor depending on the type of the parameters distribution.

Table 3: Parameters of probability distributions in case scenario Nor_Log

<table>
<thead>
<tr>
<th>No.</th>
<th>Name</th>
<th>Mean μ</th>
<th>σ/EF</th>
<th>Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>CCF-LK-STMBD</td>
<td>1,00E-04</td>
<td>2,00E-05</td>
<td>Normal</td>
</tr>
<tr>
<td>2</td>
<td>CCF-FS-FW3AB</td>
<td>3,50E-04</td>
<td>3,00E+00</td>
<td>LogNorm</td>
</tr>
</tbody>
</table>

The fourth case scenario Nor_Log_L is developed from Nor_Log scenario with increase of the error factor of the basic event with lognormal distribution as shown in Table 4.

Table 4: Parameters of probability distributions in case scenario Nor_Log_L

<table>
<thead>
<tr>
<th>No.</th>
<th>Name</th>
<th>Mean μ</th>
<th>σ/EF</th>
<th>Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>CCF-LK-STMBD</td>
<td>1,00E-04</td>
<td>2,00E-05</td>
<td>Normal</td>
</tr>
<tr>
<td>2</td>
<td>CCF-FS-FW3AB</td>
<td>3,50E-04</td>
<td>5,00E+00</td>
<td>LogNorm</td>
</tr>
</tbody>
</table>

The fifth case scenario Log_Nor is developed with change of the distributions between the basic events with introduction of lognormal distribution to the basic event with larger FV importance measure as shown in Table 5.

Table 5: Parameters of probability distributions in case scenario Log_Nor

<table>
<thead>
<tr>
<th>No.</th>
<th>Name</th>
<th>Mean μ</th>
<th>σ/EF</th>
<th>Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>CCF-LK-STMBD</td>
<td>1,00E-04</td>
<td>3,00E+00</td>
<td>LogNorm</td>
</tr>
<tr>
<td>2</td>
<td>CCF-FS-FW3AB</td>
<td>3,50E-04</td>
<td>3,50E-05</td>
<td>Normal</td>
</tr>
</tbody>
</table>
Last case scenario NNor_LLog was developed with introduction of normal distribution for two and lognormal distribution for other two basic events as shown in Table 9.

Table 6: Parameters of probability distributions in case scenario NNor_LLog

<table>
<thead>
<tr>
<th>No.</th>
<th>Name</th>
<th>Mean $\mu$</th>
<th>$\sigma/EF$</th>
<th>Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>CCF-LK-STEMBD</td>
<td>1.00E-04</td>
<td>1.00E-05</td>
<td>Normal</td>
</tr>
<tr>
<td>2</td>
<td>CCF-FS-FW3AB</td>
<td>3.50E-04</td>
<td>3.50E-05</td>
<td>Normal</td>
</tr>
<tr>
<td>3</td>
<td>MDP-FS-FW3A</td>
<td>6.30E-03</td>
<td>3.00E+00</td>
<td>LogNorm</td>
</tr>
<tr>
<td>4</td>
<td>MDP-FS-FW3B</td>
<td>6.30E-03</td>
<td>3.00E+00</td>
<td>LogNorm</td>
</tr>
</tbody>
</table>

Equal number of simulations $N=1.00E+06$ was used in the analysis of the developed case scenarios.

4 RESULTS

Obtained results for the analysed case scenarios are given in Table 7. In first row of Table 7 are given obtained parameters for the analysed case scenarios given in first row.

Table 7: Results for the analysed case scenarios

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Nor_Nor</th>
<th>Log_Log</th>
<th>Nor_Log</th>
<th>Nor_Log_L</th>
<th>Log_Nor</th>
<th>NNor_LLog</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode</td>
<td>2.97E-04</td>
<td>2.52E-04</td>
<td>2.90E-04</td>
<td>2.92E-04</td>
<td>2.45E-04</td>
<td>2.94E-04</td>
</tr>
<tr>
<td>Median</td>
<td>2.97E-04</td>
<td>2.77E-04</td>
<td>2.96E-04</td>
<td>2.94E-04</td>
<td>2.77E-04</td>
<td>2.96E-04</td>
</tr>
<tr>
<td>Mean</td>
<td>2.97E-04</td>
<td>2.97E-04</td>
<td>2.97E-04</td>
<td>2.97E-04</td>
<td>2.97E-04</td>
<td>2.97E-04</td>
</tr>
<tr>
<td>5 percentile</td>
<td>2.64E-04</td>
<td>2.24E-04</td>
<td>2.60E-04</td>
<td>2.57E-04</td>
<td>2.24E-04</td>
<td>2.76E-04</td>
</tr>
<tr>
<td>Skew</td>
<td>6.89E-01</td>
<td>2.28E+00</td>
<td>1.08E+00</td>
<td>1.96E+00</td>
<td>2.27E+00</td>
<td>1.28E+00</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>1.95E+00</td>
<td>7.04E+00</td>
<td>2.66E+00</td>
<td>5.47E+00</td>
<td>7.04E+00</td>
<td>3.14E+00</td>
</tr>
</tbody>
</table>

Obtained results in Table 7 show that for Nor_Nor case Scenario probability density function can be well approximated with normal distribution. For Log_Log case scenario, as shown in Table 7, different values are obtained for mode, median and mean of the top event probability distribution. Large value of Kurtosis obtained for Log_Log case scenario indicates heavier tails and a higher peak than the normal distribution. Table 7 show that comparable values of mode, median and mean are obtained for Nor_Log case scenario. The obtained kurtosis show that top event distribution has lighter tails and is flatter compared to normal distribution. Positive value of obtained skew indicates a right skewed distribution and this is the main implication of introduction of lognormal distribution for one of the basic events. Results for the Nor_Log_L case scenario show that small increase of the error factor of the basic event with lognormal distribution results in large increase of distribution skew and kurtosis. Obtained results for Log_Nor case scenario are comparable to the results obtained for Log_Log case scenario and show that top event probability distribution, when only two basic events have uncertainty distribution, depends on distribution of the basic event with larger FV importance measure. Introduction of lognormal distribution for two basic events that have smaller FV importance measure compared to the both events that have normal distribution, as shown in Table 7, results in increase of the difference between mode, median and mean of the probability density function compared to the scenario Nor_Nor with normal distributions. The obtained kurtosis is larger than 3 indicating that NNor_LLog case scenario will have heavier tails compared to normal distribution.

The cumulative distribution functions for all analyzed case scenarios are given on Figure 2. The cumulative distribution function for Log_Nor case scenario is equal to the one for Log_Log and is not included on Figure 8. Horizontal lines mark values of 5 percentile (full lines) and 95 percentile (dashed lines).
Figure 2: Cumulative distribution functions

Obtained results for the analyzed case scenarios in Table 7 show that the probability density function of the top event of the analyzed fault tree depends on importance of the basic events with considered uncertainty as well as parameters characterizing their uncertainty. Introduction of lognormal distribution for uncertainty characterization of basic events in the fault tree, as shown in the obtained results, may result in positive kurtosis indicating heavy tails of probability density function. The consequence of heavy tails is increased likelihood of having top event probability larger than mean value. Figure 2 shows that largest difference between 95 percentile and 5 percentile values is obtained for Log_Nor case scenario.

5 CONCLUSIONS

The analysis of the implications of the introduction of probability distributions associated with component unavailability parameters, on the overall unavailability of the analyzed system is done and obtained results are presented. The normal and lognormal distributions are introduced as probability distributions associated with component unavailability. The model of auxiliary feedwater system of the nuclear power plant was selected as test system. Six case scenarios were developed for assessment of the implications on introduction of different probability distributions for different number and sets of components.

Obtained results for the analyzed case scenarios show that the probability density function of the top event depends on type and parameters of uncertainty distributions as well as importance of the basic events with considered uncertainty. Introduction of lognormal distribution for uncertainty characterization of basic events can result in positive kurtosis indicating heavy tails of probability density function and increased likelihood of having top event probability larger than mean value.

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