Uncertainty Analysis of Infinite Homogeneous Lead and Sodium Cooled Fast Reactors at Beginning of Life

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ABSTRACT

The objective of the present work is to estimate breeding ratio, radiation damage rate and minor actinide transmutation rate of infinite homogeneous lead and sodium cooled fast reactors. Uncertainty analysis is performed under multigroup approximation. Uncertainty in nuclear data and uncertainty in the composition of the reactors are taken into account. We use the recently released ENDF/B-VII.1 nuclear data and restrict the work to beginning of reactor life.

The adjoint sensitivity analysis is performed to calculate generalized adjoint fluxes for the quantities of interest. The generalized adjoint fluxes are used to calculate first order sensitivities of the quantities of interest to input data. The acquired sensitivities are used to propagate uncertainties in the input data to find out uncertainties in the quantities of interest.

1 INTRODUCTION

The proven uranium reserves and estimated resources are sufficient to fuel the current open fuel cycle for 250 years. The thermal reactors produce minor actinides, which are more radiotoxic than natural uranium for hundreds of thousands of years. The same amount of uranium would last for 16 000 to 19 000 years with breeder reactors and closed fuel cycle. Fast breeder reactors produce insignificant amounts of minor actinides and could also transmute the presently accumulated minor actinides. Compared to thermal reactors the materials in fast reactors suffer from high radiation damage.

Nuclear data is less well known in the neutron spectrum of fast reactors than in the thermal region. The distribution of our subjective knowledge of the nuclear data can be interpreted according to the Bayesian probability interpretation. Recent evaluated nuclear data files contain rather complete sets of uncertainty estimates, i.e., evaluations of the second moments of the distribution of our subjective knowledge of the nuclear data. This allows us to estimate the second moments of the posterior distribution of the quantities of interest, giving us meaningful estimates of uncertainty due to our limited knowledge of nuclear data.

In addition to nuclear data also the composition of the reactor is taken to be uncertain. The idea is not new [1]. Even though not applied in this work, it should be noted that calculation of the sensitivities to nuclide concentrations can be used in optimization of designs against rather arbitrary performance parameters.

Since we are interested in a small number of physical quantities and the number of parameters needed to calculate them is large, it is advantageous to use adjoint sensitivity analysis, also known as generalized perturbation theory [2]. In this work its multigroup form is derived and used. The point of view is then that the multigrouped cross sections correctly represent the underlying physics. The method relies on narrow resonance approximation, which is well valid within fast energy spectrum. However, the resonance interference effect is lost.
2 THEORY

The $G$ group form of the steady state neutron criticality equation in an infinite reactor can be written as

$$\Sigma_{t,g} \Phi_g = \sum_{g' = 1}^{G} \Sigma_{r,g' \rightarrow g} \Phi_{g'} + \frac{1}{k} \sum_{g' = 1}^{G} \Sigma_{f,g' \rightarrow g} \Phi_{g'} \quad \text{for} \quad g = 1, \ldots, G, \quad (1)$$

where $k$ is the multiplication factor and $\Phi \in \mathbb{R}^G$ is the scalar neutron flux. The summation over reactions $r$ runs over all neutron emitting reactions, except fission. The macroscopic total cross section $\Sigma_{t,g}$ and the macroscopic group-to-group transfer matrices $\Sigma_{r,g' \rightarrow g}$ are defined as

$$\Sigma_{t,g} = \sum_{i=1}^{I} n_i \sigma_{i,t,g}, \quad \Sigma_{r,g' \rightarrow g} = \sum_{i=1}^{I} n_i \sigma_{i,r,g' \rightarrow g}, \quad (2)$$

where $n_i$ is the number density for the $i^{th}$ nuclide and the microscopic transfer matrix can be written in the form

$$\sigma_{i,r,g' \rightarrow g} = m_{i,r,g'} \sigma_{i,r,g' \rightarrow g}, \quad (3)$$

where $\sigma_{i,r,g}$ is the cross section of the reaction, $m_{i,r,g'}$ is the multiplicity and $p_{i,r,g' \rightarrow g}$ is the energy group distribution of the emitted neutrons. Specifically for fission $r = f$ and $m_{i,f,g'} = \bar{\nu}_{i,p,g'} + \bar{\nu}_{i,d,g'}$ is the sum of average number of prompt and delayed neutrons emitted per fission.

Reaction rates are acquired by taking the standard $\mathbb{R}^G$ inner product between a macroscopic cross section $\Sigma_r \in \mathbb{R}^G$ and the scalar flux:

$$\langle \Sigma_r, \Phi \rangle = \Sigma_r^\top \Phi, \quad (4)$$

where $\top$ denotes the transpose of a vector. The macroscopic cross section in Eq. (4) is arbitrary. In fact there are many situations where it may be zero for some groups and it may also include consequences of a reaction and not just the reaction rate, e.g., heat deposition rate or radiation damage rate.

2.1 Model parameters

The parameters in the model are nuclide concentrations $n_i$, cross sections $\sigma_{i,r,g}$, multiplicities $m_{i,r,g}$ and energy distributions of emitted neutrons $p_{i,r,g' \rightarrow g}$. The parameters include any cross sections and nuclide concentrations that are needed to calculate the responses. Together these $N$ parameters can be collected to a vector, denoted by $\alpha \in \mathbb{R}^N$.

Our subjective knowledge of the nuclide concentrations and multigroup nuclear data can be represented as a joint probability

$$p(\alpha_1, \ldots, \alpha_N) d\alpha_1 \cdots d\alpha_N \quad (5)$$

that the true value of each parameter $\alpha_n$ lies in range $(\alpha_n, \alpha_n + d\alpha_n)$ for each $n = 1, \ldots, N$ simultaneously. The probability distribution is denoted by $p(\alpha)$.

Current nuclear data files contain enough information to process the first moments and second (central) moments of the nuclear data to multigroup form. Assuming that we can estimate the first and second moments of the nuclide concentrations we have the first $E(\alpha)$ and the second moments $\text{cov}(\alpha, \alpha)$ of the probability distribution:

$$E(\alpha) = \langle \alpha \rangle \quad \text{and} \quad (6a)$$

$$\text{cov}(\alpha, \alpha) = \langle (\alpha - E(\alpha))(\alpha - E(\alpha)) \rangle, \quad (6b)$$

Here the brackets $\langle \cdot \rangle = \int \cdots \int \cdot p(\alpha) d\alpha$ indicate integration over the probability distribution.
2.2 State variables

The state variables in the model are the components of the neutron flux $\Phi$. The state variables depend on the parameters through Eq. (1). Concatenation of the state variables and parameters is denoted by $e = (\Phi, \alpha) \in \mathbb{R}^{G+N}$.

2.3 Nominal values

The nominal values $\hat{\alpha}$ of the parameters are chosen to be the first moments $E(\alpha)$. The nominal values of the state variables $\hat{\Phi}$ can be calculated by solving Eq. (1) using the nominal values of the parameters. The concatenation of the nominal values of the state variables and parameters is denoted by $\hat{e} = (\hat{\Phi}, \hat{\alpha}) \in \mathbb{R}^{G+N}$.

2.4 Responses

In this work we are interested in $L$ ratios of reaction rates, giving us responses of the type

$$R_l(e) = \frac{\langle \Sigma r, \Phi \rangle}{\langle \Sigma p, \Phi \rangle}.$$  

In principle we are interested in the full posterior distribution $p(R(e))$ for the responses $R \in \mathbb{R}^L$. Alas, since we have not fully characterized our knowledge of the parameters we can not calculate the full posterior distribution.

We can, however, approximate its first moment by linearizing the response at the nominal values of the parameters. The linearized response is

$$R_l(e) = R_l(\hat{e}) + S_l(\hat{e})(\alpha - \hat{\alpha}) + O(\|\alpha - \hat{\alpha}\|^2),$$  

where the sensitivity $S_l(\hat{e})$ accounts for both direct effects, due to variations in the parameters, and indirect effects, due to variations in the state variables which arise because of the variations in the parameters. Applying $\langle \cdot \rangle$ to both sides of Eq. (8) gives

$$E(R_l(e)) = R_l(\hat{e}) + O(\|\alpha - \hat{\alpha}\|^2),$$  

since the nominal values of the parameters were chosen to be the first moments. The result is that the first moments of the responses are approximately the nominal responses. The approximation is accurate up to second order in parameters. [4]

2.5 Uncertainty analysis

The purpose of uncertainty analysis is to estimate the higher moments of the responses. Usually it suffices to calculate the second (central) moments. An approximation for the second moments can be acquired by integrating $(R_l(e) - E(R_l(e)))(R_m(e) - E(R_m(e)))$ over the distribution of our subjective knowledge of the parameters. This gives

$$\text{cov}(R_l(e), R_m(e)) = S_l(\hat{e}) \text{cov}(\alpha, \alpha) S_m^\top(\hat{e}) + O(\|\alpha - \hat{\alpha}\|^3),$$  

which is also known as the sandwich formula. The approximation is accurate up to third order in parameters. [4]
2.6 Sensitivity analysis

A general tool for the local sensitivity analysis is the directional derivative called Gâteaux variation \([5]\). The sensitivity of a response \(R\) to variations in the parameters and state variables \(h = (h_\Phi, h_\alpha) \in \mathbb{R}^{G+N}\), respectively, at the point \(\hat{e} \in \mathbb{R}^{G+N}\) can be defined to be
\[
\delta R(\hat{e}; h) = \lim_{t \to 0} \frac{1}{t} (R(\hat{e} + th) - R(\hat{e})).
\]
For the reaction rates response considered in this work the sensitivities can be represented as
\[
d_\Phi R_l(\hat{e}; h) = R_\Phi(\hat{e}) h_\Phi + R_\Phi^\alpha(\hat{e}) h_\alpha = \langle \nabla_\Phi R_l(\hat{e}), h_\Phi \rangle + R_\alpha^\alpha(\hat{e}) h_\alpha
\]
where \(R_\Phi^\alpha\) and \(R_\alpha^\alpha\) are the gradients of the response with respect to variations in the state variables and parameters, respectively. From the viewpoint of uncertainty analysis it is the purpose of the sensitivity analysis to map \(h_\Phi\) to \(h_\alpha\), in first order, so Eq. (12) can be expressed linearly in \(h_\alpha\). This allows construction of the sensitivity as used in Eq. (8).

In operator form Eq. (11) can be written as
\[
A(\alpha)\Phi = \frac{1}{k(e)} B(\alpha)\Phi.
\]
where \(A : \mathbb{R}^{G+N} \to \mathbb{R}^G\) and \(B : \mathbb{R}^{G+N} \to \mathbb{R}^G\) are linear in \(\Phi\) but non-linear in \(\alpha\).

The adjoint system \([5]\) of Eq. (13) is defined by the identity
\[
\langle \Phi^\dagger, \left( A(\alpha) \frac{1}{k(e)} B(\alpha) \right) \Phi \rangle = \langle \left( A^\dagger(\alpha) \frac{1}{k(e)} B^\dagger(\alpha) \right) \Phi^\dagger, \Phi \rangle,
\]
where \(\Phi^\dagger \in \mathbb{R}^G\) is an adjoint flux. The associated bilinear form vanishes because boundary of the phase space is an empty set.

The forward sensitivity system \([5]\) of Eq. (13) is defined as
\[
A(\hat{e}) h_\Phi + A^\alpha(\hat{e}) h_\alpha = \frac{1}{k(e)} B(\hat{e}) h_\Phi + \frac{1}{k(e)} B^\alpha(\hat{e}) h_\alpha - \frac{1}{k(e)} \delta k(\hat{e} ; h) B(\alpha) \Phi,
\]
which can be derived by taking Gâteaux variations at \(\hat{e}\) on both sides and noticing that in this case the derivatives reduce to Jacobians.

The generalizes adjoints \(\Gamma^\dagger_l\) for the responses \(R_l\) are defined as
\[
(A^\dagger(\alpha) \frac{1}{k(e)} B^\dagger(\alpha)) \Gamma^\dagger_l = \nabla_\Phi R_l(\hat{e}) = R_l(\hat{e}) \left( \frac{\Sigma_{\Gamma_l}}{\Sigma_{\Gamma_l}} \Phi - \frac{\Sigma_{p_l}}{\Sigma_{p_l}} \Phi \right),
\]
where the operator \(A^\dagger - \frac{1}{k} B^\dagger\) will be evaluated at the nominal values of the parameters and is therefore singular. The condition \(\langle \nabla_\Phi R_l(\hat{e}), \Phi \rangle = 0\) is necessary for a solution to exist, but the reaction rate ratios do fulfill this condition. \([6]\) The general solution to Eq. (16) is \(\Gamma^\dagger_l = a \Phi^\dagger + \Gamma^\dagger_{l,p}\), where \(a \in \mathbb{R}\), the fundamental adjoint \(\Phi^\dagger\) is the solution to corresponding homogeneous equation and \(\Gamma^\dagger_{l,p}\) is the particular solution to the inhomogeneous equation. It proves to be advantageous to pick the solution which is orthogonal to the fission source, i.e., \(\langle \Gamma^\dagger_l, B(\alpha) \Phi \rangle = 0\).

Finally applying Eq. (16) to Eq. (12) and using the adjoint property of the operators, the forward sensitivity system and the orthogonality of the generalizes adjoint to the fission source gives
\[
\delta R_l(\hat{e}; h) = - \langle \Gamma^\dagger_l, \left( A^\alpha(\hat{e}) - \frac{1}{k(e)} B^\alpha(\hat{e}) \right) h_\alpha \rangle + R_\alpha^\alpha(\hat{e}) h_\alpha,
\]
which is linear in \(h_\alpha\) and can thus be used to construct \(S_l(\hat{e})\) of Eq. (8).
3 CALCULATIONS

The theory is applied to two cases: a lead and sodium cooled fast reactor at BOL.

The LFR case corresponds to volume averaged hexagonal pin cell lattice with pellet radius of 0.330 cm, gap outer radius of 0.340 cm, cladding outer radius of 0.455 cm and pin pitch of 1.365 cm.

The fuel is typical mixed oxide fuel, but contains 3.8% americium. The fuel composition is listed in table 1. Its smeared density is 9.435 g/cm³ at 1500 K. The cladding is an approximation of T91 steel at 900 K with density of 7.87 g/cm³. Isotopes of iron and chromium are included. The coolant is natural lead at 600 K. Its density is 10.66 g/cm³.

The SFR case corresponds to volume averaged hexagonal pin cell lattice with pellet radius of 0.300 cm, gap outer radius of 0.310 cm, cladding outer radius of 0.345 cm and pin pitch of 0.828 cm.

The fuel is the same as with the LFR but the cladding is 15-15Ti steel at 900 K with density of 7.87 g/cm³. It contains iron, chromium and nickel. The coolant is 23Na with 0.821 g/cm³ at 600 K.

The relative number density uncertainties correspond to 0.1% uncertainties for fuel density, 1% uncertainties for coolant and cladding densities and 1% uncertainty for weight fractions without any correlation. Strictly speaking this is incorrect since the weight fractions are constrained by sum of unity, implying that the number densities are correlated.

The quantities of interest are breeding ratio, denoted by \( R_{c,hot}/R_{a,hot} \); ratio of damage energy deposition rate in cladding to heat deposition rate, denoted by \( R_{dama}/R_{heat} \) and ratio of 241Am transmutation rate to heat deposition rate, denoted by \( R_{241}/R_{heat} \). The latter two will be referred as damage rate and transmutation rate, respectively. The transmutation rate is here defined to be sum of 241Am capture and fission rates, although captures leave some 242Am to be fissioned. The absorption rate of fissile materials in the breeding ratio is approximated as sum of their capture and fission rate.

The relevant nuclear data was converted into multigroup [7] form using NJOY [8,9] nuclear data processing system. The recently released nuclear data file ENDF/B-VII.1 [10] was used due its rather complete set of covariances. Energy distributions were assumed to be exact and their covariances were not processed. The ECCO 1968 group structure was used.

The processed correlation matrices for 207Pb and 208Pb were not positive semidefinite, which caused unphysical, negative, variances for the results. These were omitted from the calculations. The problem is most apparent with correlation matrix of total and elastic scattering cross sections.

The calculated first moments were compared to Monte Carlo code Serpent [11] to assess modeling error caused by multigroup discretization. NJOY was used to produce an ACE library for Serpent with \( 5 \times 10^{-4} \) relative tolerance. NJOY had trouble processing 204,208Pb into dosimetry ACE form and Serpent had trouble reading the dosimetry file of 206Pb. These were left out of the total heat deposition for Serpent, but were used in the transport cycles.

4 RESULTS

The values of the responses are listed in table 2. Both reactors have breeding ratio close to unity, allowing them to run by consuming almost only 235U. However, the infinite reactor does not account

<table>
<thead>
<tr>
<th>Nuclide</th>
<th>Number density (1/cm-barn)</th>
<th>Uncertainty (1/cm-barn)</th>
<th>Nuclide</th>
<th>Number density (1/cm-barn)</th>
<th>Uncertainty (1/cm-barn)</th>
</tr>
</thead>
<tbody>
<tr>
<td>238Pu</td>
<td>1.0540 \times 10^{-4}</td>
<td>1.0593 \times 10^{-6}</td>
<td>235U</td>
<td>4.7284 \times 10^{-5}</td>
<td>4.7520 \times 10^{-7}</td>
</tr>
<tr>
<td>239Pu</td>
<td>2.7336 \times 10^{-3}</td>
<td>2.7472 \times 10^{-5}</td>
<td>238U</td>
<td>1.5516 \times 10^{-2}</td>
<td>1.5593 \times 10^{-4}</td>
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<tr>
<td>240Pu</td>
<td>1.1997 \times 10^{-3}</td>
<td>1.2057 \times 10^{-5}</td>
<td>241Am</td>
<td>8.9569 \times 10^{-4}</td>
<td>9.0016 \times 10^{-6}</td>
</tr>
<tr>
<td>241Pu</td>
<td>1.8103 \times 10^{-4}</td>
<td>1.8193 \times 10^{-6}</td>
<td>16O</td>
<td>4.1917 \times 10^{-2}</td>
<td>4.2126 \times 10^{-4}</td>
</tr>
<tr>
<td>242Pu</td>
<td>3.3351 \times 10^{-4}</td>
<td>3.3517 \times 10^{-6}</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Composition of the fuel of the infinite LFR and SFR.
Table 2: LFR and SFR responses.

<table>
<thead>
<tr>
<th>Case</th>
<th>Response</th>
<th>Unit</th>
<th>Multigroup</th>
<th>Monte Carlo</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Best-estimate</td>
<td>Uncertainty</td>
</tr>
<tr>
<td>LFR</td>
<td>$R_{c,\text{fer}}/R_{a,\text{fis}}$</td>
<td>-</td>
<td>1.0575</td>
<td>0.0696</td>
</tr>
<tr>
<td>LFR</td>
<td>$R_{d,\text{ame}}/R_{\text{heat}}$</td>
<td>1/MeV</td>
<td>1.1342 $\times 10^{-3}$</td>
<td>0.1402 $\times 10^{-3}$</td>
</tr>
<tr>
<td>LFR</td>
<td>$R_{241}/R_{\text{heat}}$</td>
<td>1/MeV</td>
<td>1.4939 $\times 10^{-3}$</td>
<td>0.1398 $\times 10^{-3}$</td>
</tr>
<tr>
<td>SFR</td>
<td>$R_{c,\text{fer}}/R_{a,\text{fis}}$</td>
<td>-</td>
<td>9.8226 $\times 10^{-1}$</td>
<td>0.8795 $\times 10^{-1}$</td>
</tr>
<tr>
<td>SFR</td>
<td>$R_{d,\text{ame}}/R_{\text{heat}}$</td>
<td>-</td>
<td>4.4633 $\times 10^{-4}$</td>
<td>0.5457 $\times 10^{-4}$</td>
</tr>
<tr>
<td>SFR</td>
<td>$R_{241}/R_{\text{heat}}$</td>
<td>1/MeV</td>
<td>1.3445 $\times 10^{-3}$</td>
<td>0.1623 $\times 10^{-3}$</td>
</tr>
</tbody>
</table>

* The relative statistical error in MC is less than $10^{-4}$.

for any leakage. The softer spectrum of the SFR is seen on the radiation damage energy rate, which is about third of the LFR value. The LFR is marginally more effective in transmutation.

The modeling error, as compared to essentially exact result from Serpent, is quite low compared to the uncertainties in the input data. The maximum relative error of the responses in the LFR case is 1.1 % for damage energy rate, when heat deposition in $^{204,206,208}$Pb is omitted in both multigroup and Serpent calculations. The maximum relative error for the SFR case is 1.2 % for the damage energy rate. The other errors are less than 1 %.

4.1 LFR spectrum and generalized adjoints

The LFR neutron spectrum and generalized adjoints corresponding to the responses are shown in Fig. 1. The neutron spectrum is typical with visible flux depression for energies in large resonances of $^{56,56}$Fe and $^{16}$O. The generalized adjoints can be interpreted as importances of neutrons for the corresponding response.

The generalized adjoint of breeding ratio is negative above 1.25 MeV, corresponding to the $^{238}$U fission threshold and decrease of its capture cross section. Breeding ratio increases for every neutron introduced to the energy range 0.7 keV to 630 keV. The generalized adjoint is also positive between 0.5 eV and 7.6 eV, caused by large resonances of fertile nuclei. However, this has little practical interest for the fast reactors.

Fast neutrons deposit the most radiation damage energy: above 1.7 MeV the generalized adjoint is positive and below 0.7 MeV it is negative. The change in sign occurs because lower energy neutrons cause more heat deposition than radiation damage. Between 1 keV and 0.7 MeV the softer the

Figure 1: LFR flux and relative generalized adjoints for the considered responses.

Figure 2: Selected sensitivity profiles for LFR damage rate.
The generalized adjoint of transmutation is qualitatively similar to breeding ratio. However, the low energy peaks correspond to absorption resonances of $^{241}$Am.

### 4.2 Sensitivity profiles for the LFR damage rate

Fig. 2 shows selected sensitivity profiles for the LFR damage energy rate. The damage rate is highly sensitive to changes in $^{56}$Fe elastic cross section between 1 keV and 1 MeV. An increase in it will cause damage rate to be reduced. Physically this results from softer spectrum, which would be if the elastic scattering would be increased. Note that an increase in elastic cross section effectively decreases absorption, because as a model parameter the total cross section is held constant. The absolute value of the sensitivity has a peak near the 27.7 keV resonance of $^{56}$Fe and has a local minimum in the 24.4 keV window of $^{56}$Fe. The sensitivity profile of total cross section qualitatively mirrors the one of elastic scattering cross section.

The sensitivity to elastic scattering of $^{16}$O is negative between 1 keV and 1 MeV as $^{56}$Fe. However, the local minimum and maximum in 27.7 keV and 24.4 keV are reversed for $^{16}$O even though the cross sections for oxygen are smooth in the region. This occurs because the flux is depressed in the resonance and peaked in the cross section window. Again, the sensitivities to total cross section mostly mirror the elastic cross section.

The damage rate is much less sensitive to elastic and inelastic cross sections of $^{239}$Pu. In this case the generalized adjoint is so smooth that its effect is hard to see. The clearest effect comes from the reversal of sensitivities to total and elastic cross section above 1.7 MeV: an increase in elastic scattering in the region causes a reduction in radiation damage.

### 4.3 Uncertainty analysis

The uncertainties of the responses are quite high. The LFR damage rate is known worst with 12.3% relative standard deviation and LFR breeding ratio is known best with 6.6% relative standard deviation. The main contributions to LFR damage rate are listed in Table 3.

<table>
<thead>
<tr>
<th>#</th>
<th>Parameter pair</th>
<th>Relative total sensitivities* (-)</th>
<th>Contribution (-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$^{56}$Fe(n,elastic)</td>
<td>$^{56}$Fe(n,elastic)</td>
<td>1.8084 × 10⁻¹</td>
</tr>
<tr>
<td>2</td>
<td>$^{56}$Fe(n,elastic)</td>
<td>$^{56}$Fe(n,elastic)</td>
<td>1.7820 × 10⁻¹</td>
</tr>
<tr>
<td>3</td>
<td>$^{56}$Fe(n,total)</td>
<td>$^{56}$Fe(n,elastic)</td>
<td>1.7820 × 10⁻¹</td>
</tr>
<tr>
<td>4</td>
<td>$^{56}$Fe(n,total)</td>
<td>$^{56}$Fe(n,total)</td>
<td>1.7610 × 10⁻¹</td>
</tr>
<tr>
<td>9</td>
<td>$^{16}$O(n,elastic)</td>
<td>$^{16}$O(n,elastic)</td>
<td>6.5454 × 10⁻³</td>
</tr>
<tr>
<td>13</td>
<td>$^{16}$O(n,total)</td>
<td>$^{16}$O(n,total)</td>
<td>6.1781 × 10⁻³</td>
</tr>
<tr>
<td>19</td>
<td>$^{52}$Cr(n,elastic)</td>
<td>$^{52}$Cr(n,elastic)</td>
<td>8.4719 × 10⁻⁴</td>
</tr>
<tr>
<td>24</td>
<td>$^{52}$Cr(n,total)</td>
<td>$^{52}$Cr(n,total)</td>
<td>4.7426 × 10⁻⁴</td>
</tr>
<tr>
<td>44</td>
<td>$^{56}$Fe†</td>
<td>$^{56}$Fe†</td>
<td>8.3809 × 10⁻⁵</td>
</tr>
<tr>
<td>54</td>
<td>$^{239}$Pu†</td>
<td>$^{239}$Pu†</td>
<td>2.6430 × 10⁻⁵</td>
</tr>
</tbody>
</table>

* Integral of relative sensitivity profile over all energies.
† Number density of the nuclide.

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tribution to uncertainties of all responses. Without oxygen the relative standard deviations for LFR breeding ratio, damage rate and transmutation rate would be 3.1%, 4.1% and 4.0%, respectively. For SFR these would be 4.1%, 5.0% and 4.8% in the same order.

The contribution of uncertainties in nuclide concentrations are mostly small. The highest contributions are $^{239}$Pu with 0.14% contribution to SFR breeding ratio and $^{241}$Am with 0.07% contribution for the SFR transmutation rate. That is, they have impact on the last few printed digits of the uncertainties. The uncertainty caused by nuclide density uncertainties is likely higher in high burnup fuels. However, correlations might diminish the effect as with cross sections.

Most contributions to uncertainties are indirect. This might be partly result of the infinite homogeneous model which has only nuclear data dependency but no geometrical contributions, e.g., leakage. In the case of damage rate it results from the fact that there were no covariances for damage and heat energy deposition cross sections.

5 CONCLUSIONS

Uncertainty analysis of infinite lead and sodium cooled fast reactors has been performed by treating multigroup constants as correct representation of the underlying physics. The uncertainties in the quantities of interest are rather large due to uncertainties in the ENDF/B-VII.1 nuclear data. The impact of uncertainties in nuclide concentrations was small, but is potentially meaningful for high burnup fuels.

The low-fidelity evaluation of $^{16}$O is problematic because it contains no correlation for total and elastic reactions in covariances. This is unphysical because any increase in elastic cross section also increases the total cross section. This should be corrected in future evaluations since $^{16}$O is commonly found in many reactors.

The processed covariance matrices of $^{207}$Pb and $^{208}$Pb had negative eigenvalues, i.e., they are not proper covariance matrices. This might be a problem with the evaluation or the processing code.

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REFERENCES