Simulations of the Near-Wall Heat Transfer at Medium Prandtl Numbers

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ABSTRACT

A heat transfer from a wall to a fluid at low Reynolds and Prandtl numbers can be described by means of Direct Numerical Simulation (DNS). At higher Prandtl numbers (Pr > 20) so-called under-resolved DNS can be performed to carry out turbulent heat transfer. Three different under-resolved DNSs of the fully developed turbulent flow in the channel at Reynolds number Re = 4580 and at Prandtl numbers Pr = 100, Pr = 200 and Pr = 500 are presented in this paper. These simulations describe all velocity scales, but they are not capable to describe smallest temperature scales. However, very good agreement of heat transfer coefficients was achieved with the correlation of Hasegawa [1] or with the correlation of Papavassiliou [2], who performed DNS by means of Lagrangian method instead of Eulerian method, which was applied in our simulations. We estimate that under resolved DNS simulations based on Eulerian method are useful up to approximately Pr = 200, whereas at Pr = 500 instabilities appear due to the unresolved smallest thermal scales.

1 INTRODUCTION

Over the past 15 years Direct Numerical Simulation (DNS) has become an important research tool in understanding of the near-wall turbulent heat transfer. A DNS means precise solving of Navier-Stoke’s equations without any extra turbulent models. The first DNS of velocity field at low Reynolds number were performed by Kim and Moin [3], who investigated velocity field and observed turbulent structures in the channel. Later, they [4] also added energy equation to the equations of the velocity field for the heat transfer calculations. All these simulations were performed at low Reynolds and Prandtl numbers. Later, Kawamura et al. [5], Na and Hanratty [6] raised the limit of Prandtl number to ten, while Kawamura et al. [7] studied the influence of Reynolds numbers (up to Reτ = 395) and Prandtl numbers (up to Pr = 0.71). He found weak influence of Reynolds number and stronger influence of Prandtl number near the wall for turbulent heat transfer (velocity profiles, velocity fluctuations, turbulent heat fluxes).

Theoretically, the grid spacing for DNS of heat transfer based on Eulerian method at Prandtl numbers higher than one should be inversely proportional to the square root of Prandtl number [8]. This requirement was taken into account in the DNS studies of Kawamura et al. [5] and Tiselj et al. [9, 10]. However, as shown by Na et al. [6] and Bergant [11] this requirement is too stringent.

Papavassiliou and Hanratty [12] performed DNS of turbulent channel flow for Prandtl or Schmidt numbers that span five orders of magnitude (up to 2400). They described temperature or concentration fields without increasing resolution of numerical grid. This is possible due to Lagrangian method whose idea is based on the system of reference that moves
with heat or mass markers. In other words, heat markers, which are released from the infinitesimal source on the wall, are monitored in space and time as they move in hydrodynamic field created by DNS.

The focus of this paper was to describe the temperature field at low Reynolds number, \( \text{Re} = 4580 \), and at higher Prandtl numbers, \( \text{Pr} = 100 \), \( \text{Pr} = 200 \) and \( \text{Pr} = 500 \) (see table 1), where all temperature scales smaller than Kolmogorov scale [8] were neglected. Such numerical simulations were named as under-resolved DNS.

2 EQUATIONS AND NUMERICAL PROCEDURE

The top and bottom walls of the channel (Fig. 1) are heated by a constant heat source, while the pressure gradient drives the fluid flows between them. The flow in the channel is assumed to be fully developed. The dimensionless Navier-Stokes equations normalized by channel half height \( h \), friction velocity \( u_t = \sqrt{\nu/\rho} \), and friction temperature \( T_t = q_n/\left(\nu u_t c_p\right) \) were used. \( \tau_w \) stands for wall shear stress defined as \( \tau_w = -\mu(du/dy)_w \). Such scaling and dimensionless equations can be found in the papers of Kasagi et al. [13] or Kawamura et al. [5]:

\[
\nabla \cdot \tilde{u}^* = 0 \\
\frac{\partial \tilde{u}^*}{\partial t} = -\nabla \cdot (\tilde{u}^* \tilde{u}^*) + \frac{1}{\text{Re}_t} \nabla^2 \tilde{u}^* - \nabla p + \tilde{f} \\
\frac{\partial \tilde{\theta}^*}{\partial t} = -\nabla \cdot (\tilde{u}^* \tilde{\theta}^*) + \frac{1}{\text{Re}_t \text{Pr}} \nabla^2 \tilde{\theta}^* + \frac{u_s^*}{u_B^*}
\]

Terms \( \tilde{f} \) (unit vector in streamwise direction) and \( u_s^*/u_B^* \) appear in the equations (2) and (3) due to the numerical scheme which requires periodic boundary conditions in streamwise and spanwise directions. \( \text{Re}_t \) is friction Reynolds number and is defined as

\[
\text{Re}_t = \frac{u_t h}{\nu}
\]

where \( h \) is channel half height. The friction Reynolds number \( \text{Re}_t \) should not be confused with the usual Reynolds number, which is defined as \( \text{Re} = u_B \cdot 2h/\nu \). The usual Reynolds number in the channel can be obtained from the friction Reynolds number multiplied by the double bulk velocity \( u_B \). Friction Reynolds number \( \text{Re}_t = 150 \) corresponds to the usual Reynolds number \( \text{Re} = 4580 \). The Prandtl number is defined as
\[ \Pr = \frac{\nu}{\alpha} \]  

(5)

where \( \alpha \) is the thermal diffusivity. If we want to perform mass transfer instead of heat transfer, we simply change temperature \( \theta \) with concentration \( C \), and Prandtl number \( \Pr \) with Schmidt number \( \Sc \) in Eq. (3) [1]. The Schmidt number is defined as

\[ \Sc = \frac{\nu}{D} \],  

(6)

where \( \nu \) is kinematic viscosity and \( D \) diffusivity. Due to the similarity of the equations, results of mass and heat transfer can be compared.

Dimensionless wall units denoted with superscript + are based on the friction Reynolds number. By definition, the height of the channel is equal to two times of friction Reynolds number. The meaning of the wall units is in comparison of the turbulent flows near the wall at different Reynolds numbers. Dimensionless wall temperature difference is defined as

\[ \theta^+ (x,y,z,t) = \left( \frac{T_w - T(x,y,z,t)}{T_c} \right) \],  

(7)

The velocity components at the interface of wall and fluid are set to zero (no-slip boundary condition). Beside velocity boundary conditions, two different thermal boundary conditions can be applied at the wall-fluid interface. First, which is not presented in this paper, is isoflux boundary condition [10], and second isothermal boundary condition prescribed as

\[ \theta^+ (y = y_w) = 0 \].  

(6)

As can be seen from Eqs. (1-3) temperature is assumed to be a passive scalar. This assumption introduces two approximations: 1) neglected buoyancy, 2) neglected temperature dependence of the material properties - especially viscosity and heat conductivity. It should be emphasized that the same approximations are valid for a large number of DNS studies performed by different researchers: Kasagi et. al. [13], Kawamura et al. [5], Na and Hanratty [6], and Tiselj et al. [9]. The equations are solved with pseudo-spectral scheme using Fourier series in \( x \) and \( z \) directions and Chebyshev polynomials in the wall-normal \( y \) direction. Numerical procedure and the code of Gavrilakis [14] is used to solve the continuity, momentum and energy equations. Equations (1-3) are periodic in streamwise (\( x \)) and spanwise (\( y \)) directions.

<table>
<thead>
<tr>
<th>Table 1: Computational conditions at different Pr and grids:</th>
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<tbody>
<tr>
<td>geometry</td>
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<td>channel</td>
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The computational domain of all simulations was \( 5\pi \times 1.25\pi \times 2 \) (2356 x 589 x 300 wall units in \( x, z \) and \( y \) directions). Results were averaged after the fully developed turbulent flow was achieved, which means that the flow did not change from statistic point of view. Applied resolution was sufficient for the velocity and insufficient for the thermal field. Such
simulations were named as under-resolved DNS due to the unresolved smallest thermal scales at high wave number modes as already mentioned in introduction. However, results showed that even under-resolved DNS simulations with smallest thermal unresolved scales were sufficient for mean temperature profiles.

3 RESULTS

Fig. 2 shows average dimensionless temperatures at $Re_\tau = 150$ and at three different Prandtl numbers, $Pr = 100$, $Pr = 200$ and $Pr = 500$. The dimensionless temperature $\theta$ is averaged in planes parallel to the heated wall and in time. It should be emphasized that the temperature $\theta$ is a negative dimensionless difference, which means that a maximum temperature difference appear in the middle of the channel. Higher Prandtl number means higher dimensionless temperature difference between wall and the fluid in the middle of the channel. Comparison of the different Prandtl numbers with Kader’s empirical profile [15] was done. Excellent agreement was achieved in the diffusive sublayer, meanwhile major differences appear in the middle of the channel, where our numerical simulations show slightly larger temperature differences compared with Kader’s profile. These differences were estimated at about 6% in all three cases. Under-resolved DNS simulation at $Pr = 2400$ was also started, however numerical scheme did not converge on the applied grid, due to the very large temperature gradients near the heated wall.

![Temperature profiles for higher Prandtl numbers](image)

Figure 2. Temperature profiles for higher Prandtl numbers: Kader (curves without symbols) vs. under-resolved DNS (curves with symbols).

Heat flux in the channel can be determined by the help of the well-known equation [12]:

$$q_w = K(T_w - T_v).$$  \hspace{1cm} (8)

On the other hand, heat flux can be also given by the next equation:

$$q_w = -\lambda \frac{d\theta}{dy} \bigg|_{w}. \hspace{1cm} (9)$$
Furthermore, equations (8) and (9) can be used to derive the following relation

\[ K^+ = \frac{d\bar{\theta}}{Pr \frac{d\theta}{\theta_{\max}}}, \]  

(10)

where \( \bar{\theta}_{\max} = \bar{T} - \bar{T}_w. \) Fig. 3 shows heat transfer coefficient as function of Prandtl number.

Results, including some others, which are not introduced in this paper (Pr = 5, Pr = 5.4, Pr = 7 and Pr = 400), were compared with experimental correlation of Shaw and Hanratty [16]:

\[ K^+ = 0.0889 Pr^{-0.704}. \]  

(11)

The difference between heat transfer coefficient obtained by under-resolved DNS simulation and correlation of Shaw and Hanratty is approximately 5% at the highest Prandtl number (Pr = 500). We should stress that Shaw and Hanratty performed measurements of mass transfer at high Schmidt numbers (from Sc = 693 to Sc = 39300), but from mathematically point of view, mass transfer is identical to the heat transfer. Differences seen in Fig. 3 appeared due to the experimental uncertainties and fact that Eq. (11) is valid only in the range of Sc = 693 to Sc = 39300. In our case, dimensionless heat transfer coefficient, in the range of Pr \( \geq 100, \) is equal to

\[ K^+ = 0.073 Pr^{-0.68}. \]  

(12)

Very good agreement of heat transfer coefficients, in the range from Pr = 54 to Pr = 500, was achieved with the correlation of Hasegawa [1]

\[ K^+ = 0.0802 Pr^{-0.698} \]  

(13)

or with correlation of Papavassiliou [2]

\[ K^+ = 0.085 Pr^{-0.70} \]  

(14)

where the difference is less than 2%.

Hasegawa and Papavassilou performed DNS of mass transfer at very high Schmidt numbers (up to a few thousand) but they used Lagrangian method to describe temperature fields in contrast to our simulations where Eulerian method was used. Lagrangian approach enables lower resolution and faster time steps to perform numerical simulations at arbitrarily large Prandtl (Schmidt) number, however the magnitude of the statistical uncertainty is questionable.

In the wall proximity, the dimensionless temperature can be written as

\[ \bar{\theta} \cong Pr y^+ \]  

(15)

which means that the temperature derivative over the \( y^+ \) should be equal to Prandtl number. In our cases, derivatives are a little bit smaller (at Pr = 100 about 0.01%, at Pr = 200 about 0.6% and at Pr = 500 about 4.2%), which is one of the main reasons for lower values of the heat transfer coefficients. The error could be consequence of the lower resolution near the wall.
In a study of the resolution requirements the spectra are usually used to see how different length scales are presented. It is well known that turbulent flow consists of vortices of different dimensions. The largest vortices are defined by flow geometry, while the smallest ones are defined by viscous forces. High dissipation of the turbulent kinetic energy into the heat is typical for the smallest vortices. Viscous shear stress makes deformation work which transforms turbulent kinetic energy into internal energy of the fluid. In other words, larger vortices represented by lower wave number modes diffuse into smaller vortices represented by higher wave number modes in the spectrum diagrams. Energy of the smaller vortices is smaller than energy of the larger vortices, therefore spectra decrease with increasing wave number modes. Higher Prandtl number means slower decays of low wave number modes into the high wave number modes, and thus requires more detailed resolution to capture all significant thermal scales.

Fig. 4 shows streamwise spectra of temperature fluctuations for three different Prandtl numbers at two different distances from the wall ($y^+ = 5.8$ and $y^+ = 150$). It can be seen that spectra in the middle of the channel decrease faster than spectra near the wall. However, there is no pile-up phenomenon seen at high wave number modes. The “pile-up” phenomenon (spectrum starts to grow at high wave number modes) appears when the smallest scales are not properly modeled and viscous (temperature) dissipation in viscous (conductive) scale cannot change all turbulent kinetic energy into internal energy. It means that resolution should be increased in order to capture the smallest (Kolmogorov) scales. We can conclude, that the smallest temperature structures, which were not captured with the numerical simulation, do not have considerable influence on the temperature field. Fig. 4 also shows, that spectra of lower Prandtl numbers with minor part of the neglected scales, decrease faster than spectra of higher Prandtl numbers in the range of high-wave number modes.
4 CONCLUSIONS

Numerical simulations of turbulent heat transfer at friction Reynolds number, $Re_tr = 150$ and at three different Prandtl numbers $Pr = 100$, $Pr = 200$ and $Pr = 500$ were performed. Temperature was assumed to be a passive scalar. Such numerical simulations, called under-resolved DNS, describe all velocity scales, but are not able to describe all smallest thermal scales. Comparison of correlation of heat transfer coefficient with numerical correlation of Hasegawa [1] and Papavassiliou [2] show, that differences are not larger than 2%. We should stress that Hasegawa’s and Papavassiliou’s results were taken with Lagrangian approach in contrast with our results taken by Eulerian approach. Comparison of correlation of heat transfer coefficient with older empirical correlation of Shaw and Hanratty [16] and comparison of temperature profiles with Kader’s profiles show larger differences (up to 6%).

We estimate, that under-resolved DNS simulations are usefull for Prandtl numbers smaller than 200. At higher Prandtl numbers, the influence of the neglected temperature scales, become important in such a manner, that they could not be ignored.

NOMENCLATURE

$D$ diffusivity
$EE$ spectrum
$h$ channel half height
$\hat{r}$ unit vector in $x$ direction (1,0,0)
$K$ heat transfer coefficient
$k$ wave number
$L_x, L_z$ streamwise and spanwise length of box
$p$ pressure
$Pr$ Prandtl number ($Pr = \nu/\alpha$ )
$q_w$ wall-to-fluid heat flux
$Re$ Reynolds number
$x, y, z$ streamwise, spanwise, wall normal distance
$u, w, v$ velocity components in $x, y$ and $z$ directions
$u_i$ dissipative velocity
$\alpha$ thermal diffusivity
$\theta$ dimensionless temperature difference
$\lambda$ thermal conductivity
$\gamma$ kinematic viscosity
$\rho$ density
$\Omega_c$ middle of the channel
$\Omega_f$ fluid
$\Omega_t$ turbulent

Proceedings of the International Conference Nuclear Energy for New Europe, Portorož, Slovenia, Sept. 8-11, 2003
Re,

friction Reynolds number (Re,

= \frac{\nu, h}{\nu})

Ow

wall

Sc

Schmidt number (Sc = v/D)

 \( \rho \)

dissipation

T

temperature

\( \rho' \)

normalized by \( \nu' , T' , \nu \)

\( t' \)

(time fluctuations)

REFERENCES


