Numerical Simulation of the Thermal Behavior of Heat Transfer Equipment Operated at Low Temperature

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ABSTRACT

The paper presents a method for calculating the non-steady heat transfer in a shell and tube heat exchanger. The characteristic equations were solved with a Finite Element Method. As the geometry is cylindrical and axial symmetry was assumed, the equations were solved in a two dimensional geometry. The interpolation functions are linear and the Galerkin method was applied. The process occurred without phase change. For the solving of the algebraic equations associated with the differential equations, we used the method of steepest descendent (gradient method). As results, we present the temperature profile for the tube and shell gas.

1 INTRODUCTION

The simulation of heat transfer processes in specific equipments represents an area of large interest, especially in the field of cryogenic temperatures, which represents one of the characteristic fields of our research team.

The realization of experimental plants operated at liquid nitrogen temperature or below this temperature, in order to ensure the right level of temperature, require theoretical studies regarding the determination of performances and characteristics for the heat transfer in heat exchangers.

Our institute develop a detritiation technology with the purpose to remove the tritium from heavy water used like moderator in CANDU type reactors. The technology is based on a catalytic isotopic exchange, purification and cryogenic distillation. The cryogenic module proposed is composed of feed circuit, a nitrogen circuit which ensure a primary cooling and a high pressure hydrogen circuit which ensure the operation temperature of the cryogenic distillation column. The heat exchangers that presents interest for us are those used in the cryogenic distillation module. The paper represents for us a first step in numerical simulation of the thermal behavior using a Finite Element Method for the prediction of the non-stationary temperature distribution in a tube and shell heat exchanger.

Through the use of simulation techniques, more accurate analysis can be carried out. Besides heat transfer calculations, studies can be made to determine other characteristics like pressure drops or global heat transfer coefficient. The studies will be applied for testing different types of heat exchangers used in modules included in a detritiation plant, in particular in a cryogenic distillation module.
2 HEAT TRANSFER IN THE SHELL-TUBE HEAT EXCHANGER

We are interested in studying the non-steady heat transfer in a shell and tube heat exchanger-counterflow (figure 1a). Having in view the special requirements imposed for heat transfer equipment, it is important to have a precise image about the time for steady-state or temperature profile in the direction of fluid flow while the velocity profile remains constant through the length of the heat exchanger section [1].

We intend to solve the differential equation of heat transfer in fluid and wall. For this, we choose to solve the equations with a Finite Element Method.

For the wall, the conduction is characterised by the Fourier equation:

$$\rho \cdot c_p \cdot \frac{\partial T}{\partial \tau} = \lambda \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + q,$$  
(1a)

We considered that the fluid circulated in the shell and tube is hydrogen in gaseous state. We supposed that the process occurs without phase change. The equation we applied for the fluid is [2]:

$$\frac{\partial T}{\partial \tau} + v_x \cdot \frac{\partial T}{\partial x} + v_y \cdot \frac{\partial T}{\partial y} + v_z \cdot \frac{\partial T}{\partial z} = \rho \cdot c_p \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + \frac{q}{\rho \cdot c_p}.$$  
(1b)

The modelling will be presented simultaneously for the convective transfer and, also, for the conductive one, in cylindrical coordinates: (r,θ,z).
For the finite element discretization, we choose quadrilateral elements with four nodes with a system of natural coordinates [3]. In each element, the unknown function is the temperature which can be approximated as:

\[ T = T_1 \Phi_1 + T_2 \Phi_2 + T_3 \Phi_3 + T_4 \Phi_4 \]  

(2)

For solving the two equations for heat transfer (1a) and (1b), the Galerkin method was applied [4]. This method supposed that if a numerical solution of a differential equation approximate it until a residue \( \varepsilon \), then the scalar product between this residue and a set of orthogonal functions is 0. That set of functions is just the set of interpolation functions. Having in view that the spatial variation is independent from time, for our equations we can write:

\[ \int_0^\Delta \int_\Omega \{ E(T) \} \Phi dr dz \right] \Phi' dt = 0 \]  

(3)

We can solve the problem through separate integration after time and spatial coordinates, (r,z):

\[
\frac{\rho c_p}{k} \int_\Omega \left[ \Phi \left( \frac{\partial T}{\partial t} - q \right) \right] dr dz = \int_\Omega \Phi \left[ \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} + \frac{\partial^2 T}{\partial z^2} \right] dr dz
\]  

(4a)

\[
\int_\Omega \Phi \frac{\partial T}{\partial t} dr dz = \int_\Omega \Phi \left[ a \frac{\partial^2 T}{\partial r^2} + \left( \frac{a}{r} - v_r \right) \frac{\partial T}{\partial r} - v_z \frac{\partial T}{\partial z} + a \frac{\partial^2 T}{\partial z^2} \right] dr dz
\]  

(4b)

We noted with I the left hand side of the equation (4a) and with II the left hand side of the equation (4b). The next step is to rewrite the right hand side in the following manner, similarly for r and z [5]:

\[
\Phi \frac{\partial^2 T}{\partial z^2} = \frac{\partial}{\partial z} \left( \Phi \frac{\partial T}{\partial z} \right) - \frac{\partial T}{\partial z} \cdot \frac{\partial \Phi}{\partial z} 
\]  

(5)

Applying the Green theorem in both cases, we obtain:

\[
I + \int_\Omega \left[ \frac{\partial T}{\partial r} \left( \frac{\partial \Phi}{\partial r} - \frac{\Phi}{r} \right) + \frac{\partial T}{\partial z} \cdot \frac{\partial \Phi}{\partial z} \right] dr dz = \int_L \Phi \frac{\partial T}{\partial z} \, dr + \Phi \frac{\partial T}{\partial r} \, dz 
\]  

(6a)

\[
II + \int_\Omega \left[ a \frac{\partial \Phi}{\partial r} \frac{\partial T}{\partial r} - \left( \frac{a}{r} - v_r \right) \Phi \frac{\partial T}{\partial r} + v_z \Phi \frac{\partial T}{\partial z} + a \frac{\partial \Phi}{\partial z} \frac{\partial T}{\partial z} \right] dr dz = a \int_L \Phi \frac{\partial T}{\partial z} \, dr + \Phi \frac{\partial T}{\partial r} \, dz
\]  

(6b)

Finally we can write for the right part of the both equations above the following relation:

\[
\int_L \left[ - \frac{\partial T}{\partial z} \, dr + \Phi \frac{\partial T}{\partial r} \, dz \right] = \int_L \Phi \frac{\partial T}{\partial z} \cos(n, z) + \Phi \frac{\partial T}{\partial r} \cos(n, z)
\]  

(7)
The equation written in form below allowed us to introduce the boundary conditions. Analysing the conditions inside of the discretization network proposed in Figure 1b, it can be seen that for the elements which have all the sides inside the network, the left hand of the equation (6b) is zero, due to the opposite sign of the path when the integral is solved [6].

In this way, we introduced the boundary values of the problem that are:

- the outside boundary of the heat exchanger is an adiabatic surface
- the heat flux on the axis of symmetry is 0 (from symmetry reasons), that way we study the system such is presented in Figure 1
- the inlet temperatures and the flow values of the agents are maintained constant during the experiment.

In order to realise an accuracy as good as possible and an adequate speed of calculation, the equations written above and the big number of unknowns determine us to introduce in this modelling some simplifying aspects:

- the velocity of the gas is predominant in the fluid moving direction, in our case, in the z direction
- the radiation is neglected, so that we considered for heat transfer: the convection for the fluid and the conduction for the wall.

For the elements which are bordering the boundary, we have the four cases presented in the following table:

<table>
<thead>
<tr>
<th>Side</th>
<th>Cosines values</th>
<th>Contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r=ct$</td>
<td>$n=-dz$</td>
<td>$dT/dz=0$</td>
</tr>
<tr>
<td>$r=ct$</td>
<td>$n=dz$</td>
<td>$dT/dz=0$</td>
</tr>
<tr>
<td>$z=ct$</td>
<td>$n=dr$</td>
<td>$dT/dr=0$</td>
</tr>
<tr>
<td>$z=ct$</td>
<td>$n=-dr$</td>
<td>$q=dT/dr+dT/dz=0$</td>
</tr>
</tbody>
</table>

With these conditions, the equations become:

\[
\int_{\Omega} \left[ \frac{\rho c_p}{k} \left( \frac{\partial T}{\partial t} - q_v \right) + \frac{\partial T}{\partial r} \left( \frac{\partial \Phi}{\partial r} - \frac{\Phi}{r} \right) + \frac{\partial T}{\partial z} \frac{\partial \Phi}{\partial z} \right] dr dz = 0 \quad (8a)
\]

\[
\int_{\Omega} \left[ \Phi \frac{\partial T}{\partial t} + a \frac{\partial \Phi}{\partial r} \frac{\partial T}{\partial r} - \left( \frac{a}{r} - v_z \right) \Phi \frac{\partial T}{\partial r} + v_z \frac{\partial T}{\partial z} + a \frac{\partial \Phi}{\partial z} \frac{\partial T}{\partial z} \right] dr dz = 0 \quad (8b)
\]

Regarding the time variation, we supposed that the temperature in a point from the network could be written, as a function of the temperature in that point at a previous moment and the time interval, as a linear variation:

\[
T = \left(1 - \frac{t}{\Delta t} \right) T_0 + \frac{t}{\Delta t} T_f \quad (9)
\]

The partial derivative of the equation:

\[
\frac{\partial T}{\partial t} = - \frac{T_0}{\Delta t} + \frac{T_f}{\Delta t} \quad (10)
\]
With this approximation and rearranging the terms, we obtain:

\[
\int_{\Omega} T_{f} \left[ \frac{\rho c_p \Phi}{2k} + \frac{\Delta t}{3} \left( \frac{\partial \Phi}{\partial r} - \Phi \frac{\partial}{\partial r} \right) \frac{\partial}{\partial r} + \frac{\Delta t}{3} \left( \frac{\partial \Phi}{\partial z} \right) \frac{\partial}{\partial z} \right] +
\int_{\Omega} T_{o} \left[ -\frac{\rho c_p \Phi}{2k} + \frac{\Delta t}{6} \left( \frac{\partial \Phi}{\partial r} - \Phi \frac{\partial}{\partial r} \right) \frac{\partial}{\partial r} + \frac{\Delta t}{6} \left( v_z \Phi + a \frac{\partial \Phi}{\partial z} \right) \frac{\partial}{\partial z} \right] drdz = 0 \tag{11a}
\]

\[
\int_{\Omega} T_{f} \left[ \Phi + \frac{\Delta t}{3} a \left( \frac{\partial \Phi}{\partial r} - \left( \frac{a}{r} - v_r \right) \Phi \right) \frac{\partial}{\partial r} + \frac{\Delta t}{3} \left( v_z \Phi + a \frac{\partial \Phi}{\partial z} \right) \frac{\partial}{\partial z} \right] +
\int_{\Omega} T_{o} \left[ -\frac{\Phi}{2} + \frac{\Delta t}{6} a \left( \frac{\partial \Phi}{\partial r} - \left( \frac{a}{r} - v_r \right) \Phi \right) \frac{\partial}{\partial r} + \frac{\Delta t}{6} \left( v_z \Phi + \frac{\partial \Phi}{\partial z} \right) \frac{\partial}{\partial z} \right] drdz = 0 \tag{11b}
\]

These are the equations we have solved, which provided the values of the temperature in the nodes of the network. With these, we can compute the thermal field in the whole heat exchanger within the temperature profile in the direction of fluid flow.

The integration was performed using Newton algorithm. The equations establish a system of \( k \) non-linear algebraically equations with \( k \) unknowns. For solving this system we used the step descendent method.

The thermodynamic proprieties of the gas (in our case hydrogen) and the tube material are introduced in polynomial form. The functions (density, specific heat at constant pressure, conductivity) are depending on temperature, at the pressure level considered [7], [8].

3 COMPUTATIONAL RESULTS

In order to compute the temperature field, the equations (1) were solved for the geometry presented in Figure 1 with the boundary conditions and the simplifying hypothesis presented above. The fluid considered in tube and shell was hydrogen in gaseous state.

In this step of simulation we analysed a primary cooling in the temperature range [70, 280]K. We considered this temperature range because we intended to compare with a set of existing results from the literature [9].

The initialising conditions are: the inlet temperature of primary gas in tube is 280 K at a pressure of 1 bar and the inlet temperature of the secondary gas in shell is 80 K at a pressure of 3 bar. The time step was 0.1s and for each element we considered 5 steps of integration in the \( \xi \) direction and 20 steps of integration in the \( \eta \) direction.

The structure of the program is presented in Figure 2. The program provides the temperature values in the nodes of the network (see Figure 1b).

With these values we can plot temperature profile in the \( z \) direction at different moments of time. Such sets of values are represented in Figure 3. So that, we plot the temperature profile at 1 minute from the start (a), 5 minutes (b) and at 12 minutes (c).
Read initial data and system geometry

Finite element network (establish the nodes)

Coordinates crossing from \((r,z)\) to natural ones \((\xi,\eta)\)

Establish the temperature spectrum in network nodes

A. Algebraic system compound of \(VF\) equations and Jacobian \(W\)

B. Solving the equation system with gradient method \(\alpha\)

Comparison with admissible error level \(VF < \varepsilon\)

DA

Temperature field from nodes of network

Figure 2: The structure of the program

Figure 3: Temperature profile in long of heat
\(a\) – 1 min; \(b\) – 5 min; \(c\) – 12 min
4 CONCLUSIONS

A method of simulating heat exchanger is through the use of computers. In order to solve the resulting set of equations using the computer, the partial differential equations must be reduced to a set of equations using different modelling techniques. The most of applications use the finite difference techniques for the prediction of the transient temperature distribution in heat exchangers [9]. The model proposed in this paper will be utilised to simulate the heat transfer in a shell and tube heat exchanger using a Finite Element Method which represents a new approach for this problem. This technique allow us a better analysis of the process. In order to continue the investigation of the results of running program, the temperature in any point of the network at any time between 0-16 minutes may be analysed. We chosen the outlet temperature in the middle of the stream for the two gases, plot presented in Figure 4 with blue representation. Also, in Figure 4 there presented the results from literature (red representation), values obtained with a model based on the finite difference technique and the same inlet values [9].

![Figure 4: Temperature evolution in time](image-url)

The model will be applied to simulate the heat transfer for different types of heat exchangers. Further on, we intend to develop the method for a complex geometry, beside of reducing the number of simplifications as much as is possible. Also, the next step is to realise an experimental stand where it can be tested the equipment. The results obtained with the model will be compared with experimental results and the data will be available for verify the design and to test the existing heat exchangers.
REFERENCES


