Kinetic Model of a Sheath Formed in front of a Floating Collector that Emits Secondary electrons with Different Electron and Ion Emission Coefficients

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ABSTRACT

A static kinetic model of a bounded plasma system that contains energetic electrons and is limited by a planar collector that emits electrons is modified by the introduction of electron and ion emission coefficient and some preliminary results are presented. It turns out that the moments of the particle distribution functions do not depend on the ratio between the ion and the electron critical emission coefficients. When the critical emission coefficients are plotted as functions of the density of energetic electrons, it turns out that these functions have very well defined maximum values.

1 INTRODUCTION

In many plasma devices for material processing, plasmas with additional energetic electrons are readily produced. Such electron populations appear also in fusion machines due to strong rf fields during ion cyclotron and lower hybrid wave heating and rf current drive. The presence of energetic electrons has a remarkable effect on the potential formation in the plasma and consequently on particle losses to the wall. Recently we have presented [1] a one-dimensional kinetic model of a bounded plasma system that contains two maxwellian electron population with two different temperatures. On one side the plasma system is bounded by a planar source that emits singly charged positive ions, cool electrons and hot electrons. All with half-maxwellian distribution functions. On the other side the system is bounded by a floating collector that absorbs all the particles that hit it are absorbed. The collector emits secondary electrons. In the previous paper [1] only the electron emission coefficient has been included in the model. In this paper we present an extension of the previous model by including also the ion emission coefficient.
2 THE MODEL

2.1 The Particle Distribution Functions

A planar plasma source is located at \( x = 0 \). The system is bounded at \( x = L \) by a floating collector. The plasma consists of 4 species of charged particles: singly charged positive ions (index \( i \)), cool electrons (index 1), hot electrons (index 2) and secondary electrons (index 3). The ions, the cool and the hot electrons are injected from the source, all with half-maxwellian velocity distribution functions, each particle species with its own temperature and density at the source. The offset velocity of all three distribution functions at the source is assumed to be zero. The secondary electrons are emitted from the collector with half-maxwellian velocity distribution function and zero offset velocity at the collector. The potential at the source is set to zero and the condition is imposed that the electric field there must also be zero. As the electrons are more mobile than the ions, the floating collector biases negatively. We are interested in such states of the bounded plasma system, where the potential \( \Phi(x) \) decreases monotonically from the source to the collector. The plasma potential \( \Phi(x) \) is therefore negative for any \( x \). We label the floating potential of the collector as \( \Phi(x=L) = \Phi_C \). It is assumed that all the ions are absorbed at the collector. We assume that the plasma is collisionless and that the energy of particles is conserved.

The ions that are born at the source with zero velocity, have at the distance \( x \) from the source the velocity:

\[
\nu_{\text{mi}} = \sqrt{\frac{2e_i \Phi(x)}{m_i}}. \tag{1}
\]

The velocity distribution function for the ions is then written as:

\[
f_i = n_{Si} \sqrt{\frac{m_i}{2\pi kT_{Si}}} \exp\left(-\frac{e_i \Phi(x)}{kT_{Si}}\right) \exp\left(-\frac{m_i \nu^2}{2kT_{Si}}\right) H(\nu - \nu_{\text{mi}}). \tag{2}
\]

Here \( n_{Si} \) is the density and \( T_{Si} \) is the temperature that the ions would have at the source if their distribution function there was fully maxwellian. The Boltzmann constant is \( k \) and \( H \) is the Heaviside step function.

An electron that has almost reached the collector, but has then been repelled, or a secondary electron that has left the collector with zero velocity, will have at the distance \( x \) from the source the velocity

\[
\nu_{\text{me}} = -\sqrt{\frac{2e_e (\Phi(x)-\Phi_C)}{m_e}}. \tag{3}
\]

So the distribution function for the cool electrons is written as:

\[
f_i = n_{Si} \sqrt{\frac{m_e}{2\pi kT_{Si}}} \exp\left(-\frac{e_e \Phi(x)}{kT_{Si}}\right) \exp\left(-\frac{m_e \nu^2}{2kT_{Si}}\right) H(\nu - \nu_{\text{me}}). \tag{4}
\]

Here \( n_{Si} \) is the density and \( T_{Si} \) is the temperature that the cool electrons would have at the source if their distribution function there was fully maxwellian. The distribution function for the hot electrons is:

\[
f_i = n_{Si} \sqrt{\frac{m_e}{2\pi kT_{Si}}} \exp\left(-\frac{e_e \Phi(x)}{kT_{Si}}\right) \exp\left(-\frac{m_e \nu^2}{2kT_{Si}}\right) H(\nu - \nu_{\text{me}}). \tag{5}
\]

Here \( n_{Si} \) is the density and \( T_{Si} \) is the temperature that the hot electrons would have at the source if their distribution function there was fully maxwellian. The distribution function for the secondary electrons is:
\[ f_3 = n_{c3} \sqrt{\frac{m_e}{2\pi kT_{c3}}} \exp \left( \frac{e_0 \Phi(x) - \Phi_c}{kT_{c3}} \right) \exp \left( -\frac{m_e v^2}{2kT_{c3}} \right) H(v_{me} - v). \] 

Here \( n_{c3} \) is the density and \( T_{c3} \) is the temperature that the secondary electrons would have at the collector if their distribution function there was fully maxwellian.

The following variables are introduced:

\[ \mu = \frac{m_e}{m_i}, \quad \tau = \frac{T_{si}}{T_{si}}, \quad \Theta = \frac{T_{c3}}{T_{si}}, \quad \sigma = \frac{e_0 \Phi(x)}{kT_{si}}, \quad \Psi = \frac{e_0 \Phi(x)}{kT_{si}}, \quad \Psi_c = \frac{e_0 \Phi(x = L)}{kT_{si}} \]

\[ \alpha = \frac{n_{si}}{n_{si}}, \quad \beta = \frac{n_{s2}}{n_{si}}, \quad \epsilon = \frac{n_{c3}}{n_{si}}, \quad v_0 = \sqrt{\frac{2kT_{si}}{m_e}}, \quad u = \frac{v}{v_0}. \]

With these variables the distribution functions are written in the following way:

\[ F_i = \frac{\alpha}{\sqrt{\pi}} \frac{1}{\tau \mu} \exp \left( -\frac{\Psi}{\tau} \right) \exp \left( -\frac{u^2}{\mu \tau} \right) H(u - u_{mi}), \]

\[ F_1 = \frac{1}{\sqrt{\pi}} \exp(\Psi) \exp\left( u^2 \right) H(u - u_{me}), \]

\[ F_2 = \frac{\beta}{\sqrt{\pi}} \Theta \exp \left( \frac{\Psi}{\Theta} \right) \exp \left( -\frac{u^2}{\Theta} \right) H(u - u_{me}), \]

\[ F_3 = \frac{\epsilon}{\sqrt{\pi}} \frac{\Psi - \Psi_c}{\sigma} \exp \left( -\frac{u^2}{\sigma} \right) H(u_{me} - u), \]

where

\[ u_{mi} = \frac{v_{mi}}{v_0} = \sqrt{-\mu \Psi}, \quad u_{me} = \frac{v_{me}}{v_0} = \sqrt{\Psi - \Psi_c}. \]

The above distribution functions are normalized to the cool electron density at the source in the following way:

\[ n_{si} \sqrt{\frac{m_e}{2\pi kT_{si}}} \int_{-\infty}^{\infty} \exp \left( -\frac{m_e v^2}{2kT_{si}} \right) dv = \frac{1}{\sqrt{\pi}} \frac{n_{si}}{v_0} \int_{-\infty}^{\infty} \exp \left( -\frac{v^2}{v_0^2} \right) dv = n_{si} \equiv 1. \]

### 2.2 Moments of the Distribution Functions

The zero moments of the distribution functions give the normalized densities measured in the units of \( n_{si} \) and are calculated as:

\[ N_k = \int F_k(u)du. \]

Here \( k \) stands for \( i, 1, 2, \) and 3. The first moments of the distribution functions give normalized fluxes measured in the units of \( n_{si}v_0 \):

\[ J_k = \int uF_k(u)du. \]

### 2.3 Floating collector

If the collector is floating, the total flux of charged particles to the collector must be zero:

\[ J_i + J_3 = J_1 + J_2. \]
Now an assumption must be made about the electron flux \( J_3 \) which is emitted from the collector. We assume that the flux of emitted electrons is proportional to the flux of incoming electrons and ions and that the proportionality constants are different:

\[
J_3 = \gamma_e (J_1 + J_2) + \gamma_i J_i.
\]

The proportionality constant \( \gamma_e \) is called the electron emission coefficient and the proportionality constant \( \gamma_i \) is called the ion emission coefficient. From equations (13) and (14) one finds:

\[
J_i = \frac{1 - \gamma_e}{1 + \gamma_i} (J_1 + J_2).
\]

### 2.4 Relation between the plasma potential, the floating potential of the collector and the emission coefficients

The plasma potential \( \Psi(z) \) between the source and the collector is determined by the Poisson equation. Somewhere between the source and the collector (at \( z = z_0 \)) the plasma is neutral. The value of plasma potential at this point is \( \Psi(z = z_0) = \Psi_p \). At \( z = z_0 \) the Poisson equation in one dimension reads:

\[
\left( \frac{d^2\Psi}{dz^2} \right)_{z=z_0} = N_i(\Psi_P) - N_1(\Psi_P) - N_2(\Psi_P) - N_3(\Psi_P) = 0.
\]

Here \( z \) is the dimensionless space coordinate, normalized by the Debye length for the cool electrons at the source:

\[
z = \frac{x}{\lambda_D}, \quad \lambda_D = \sqrt{\frac{e_0 k T_{S1}}{n_{S1} e_0^2}}.
\]

Hence setting the net charge density to zero finds the inflection point of the plasma potential. Since:

\[
\frac{1}{2} \frac{d}{dz} \left( \frac{d\Psi}{dz} \right)^2 = \frac{d\Psi}{dz} \frac{d^2\Psi}{dz^2},
\]

equation (16) is multiplied by \( d\Psi/dz \) and integrated once over \( z \) from \( z = 0 \) to \( z = z_0 \). The differentials \( dz \) cancel each other out and integration over \( z \) is transformed into integration over \( \Psi \) in the following form:

\[
\frac{1}{2} \left( \frac{d\Psi}{dz} \right)^2 \bigg|_0^{\Psi_F} \left[ N_i(\Psi_P) - N_1(\Psi_P) - N_2(\Psi_P) - N_3(\Psi_P) \right] d\Psi = 0.
\]

The derivative \( d\Psi/dz \) is proportional to the electric field. At \( z = z_0 \) the electric field is zero. Equation (18) is therefore called zero field condition at the inflection point.

If the emission coefficients \( \gamma_i \) and/or \( \gamma_e \) increase, eventually the density of the secondary electrons in front of the collector becomes so high, that the electric field at the collector becomes zero. The values of \( \gamma_i \) and \( \gamma_e \) at which the electric field becomes zero are called the critical emission coefficients. We label them \( \gamma_i^c \) and \( \gamma_e^c \). This fact can be used to derive another expression relating \( \Psi_P \) and \( \Psi_C \). This relation is called zero field condition at the collector and it is derived in a similar way as the zero field condition at the inflection point. Only the boundaries of the integration are changed. The integration goes from \( z = z_0 \) to \( z = L \). Together with the boundary condition \( \Psi(z=L/\lambda_D) = \Psi_C \) this gives:

\[
\frac{1}{2} \left( \frac{d\Psi}{dz} \right)^2 \bigg|_{\Psi_p}^{\Psi_F} \left[ N_i(\Psi_P) - N_1(\Psi_P) - N_2(\Psi_P) - N_3(\Psi_P) \right] d\Psi = 0.
\]
When the distribution functions (8) are inserted into (11) and (12) and then (13) - (15) are used to express $\alpha$ and $\varepsilon$ in terms of $\gamma_i$ and $\gamma_e$ equation (16) becomes:

$$\frac{1 - \gamma_e}{(1 + \gamma_e) \sqrt{\mu^2}} \exp(\Psi_c) \left[ 1 + \beta \sqrt{\Theta} \exp \left( \frac{\Psi_c (1 - \Theta)}{\Theta} \right) \right] \exp \left( -\frac{\Psi_p}{\tau} \right) \text{Erfc} \left( \sqrt{-\frac{\Psi_p}{\tau}} \right) = \exp(\Psi_p) \left[ 1 + \text{Erf} \left( \sqrt{\Psi_p - \Psi_c} \right) \right] + \beta \exp \left( \frac{\Psi_p}{\Theta} \right) \left[ 1 + \text{Erf} \left( \sqrt{\Psi_p - \Psi_C} \right) \right] + \gamma_i + \gamma_e \exp(\Psi_c) \left[ 1 + \beta \sqrt{\Theta} \exp \left( \frac{\Psi_c (1 - \Theta)}{\Theta} \right) \right] \exp \left( -\frac{\Psi_p - \Psi_c}{\sigma} \right) \text{Erfc} \left( \sqrt{-\frac{\Psi_p - \Psi_C}{\sigma}} \right).$$

(20)

Using the same procedure the equation (18) (zero field condition at the inflection point becomes:

$$\frac{1 - \gamma_e}{(1 + \gamma_e) \sqrt{\mu^2}} \exp(\Psi_c) \left[ 1 + \beta \sqrt{\Theta} \exp \left( \frac{\Psi_c (1 - \Theta)}{\Theta} \right) \right] \left[ 1 - \frac{2}{\sqrt{\pi}} \sqrt{-\frac{\Psi_p}{\tau}} \exp \left( -\frac{\Psi_p}{\tau} \right) \text{Erfc} \left( \sqrt{-\frac{\Psi_p}{\tau}} \right) \right] = \exp(\Psi_p) \left[ 1 + \text{Erf} \left( \sqrt{\Psi_p - \Psi_c} \right) \right] - \frac{2}{\sqrt{\pi}} \exp(\Psi_c) \left( \sqrt{\Psi_p - \Psi_c} - \sqrt{-\Psi_C} \right) - [1 + \text{Erf} \left( -\sqrt{\Psi_C} \right) ] + \beta \Theta \exp \left( \frac{\Psi_p}{\Theta} \right) \left[ 1 + \text{Erf} \left( \sqrt{\Psi_p - \Psi_c} \right) \right] - \frac{2}{\sqrt{\pi}} \exp(\Psi_c) \left( \sqrt{\Psi_p - \Psi_c} - \sqrt{-\Psi_C} \right) - [1 + \text{Erf} \left( -\sqrt{\Psi_C} \right) ] + \frac{\gamma_i + \gamma_e}{(1 + \gamma_e) \sqrt{\sigma}} \exp(\Psi_c) \left[ 1 + \beta \sqrt{\Theta} \exp \left( \frac{\Psi_c (1 - \Theta)}{\Theta} \right) \right] \left[ 1 - \frac{2}{\sqrt{\pi}} \sqrt{-\frac{\Psi_p - \Psi_C}{\sigma}} \exp \left( \sqrt{-\frac{\Psi_p - \Psi_C}{\sigma}} \right) \text{Erfc} \left( \sqrt{-\frac{\Psi_p - \Psi_C}{\sigma}} \right) \right].$$

(21)

The zero field condition at the collector (19) becomes:

$$\frac{1 - \gamma_e}{(1 + \gamma_e) \sqrt{\mu^2}} \exp(\Psi_c) \left[ 1 + \beta \sqrt{\Theta} \exp \left( \frac{\Psi_c (1 - \Theta)}{\Theta} \right) \right] \left[ 1 - \frac{2}{\sqrt{\pi}} \sqrt{-\frac{\Psi_p}{\tau}} \exp \left( -\frac{\Psi_p}{\tau} \right) \text{Erfc} \left( \sqrt{-\frac{\Psi_p}{\tau}} \right) \right] = \exp(\Psi_c) \left[ 1 + \frac{2}{\sqrt{\pi}} \sqrt{\Psi_p - \Psi_c} \right] - \exp(\Psi_p) \left[ 1 + \text{Erf} \left( \sqrt{\Psi_p - \Psi_c} \right) \right] + \beta \Theta \exp \left( \frac{\Psi_c}{\Theta} \right) \left[ 1 + \frac{2}{\sqrt{\pi}} \sqrt{\Psi_p - \Psi_c} \right] - \exp(\Psi_p) \left[ 1 + \text{Erf} \left( \sqrt{\Psi_p - \Psi_c} \right) \right] + \frac{\gamma_i + \gamma_e}{(1 + \gamma_e) \sqrt{\sigma}} \exp(\Psi_c) \left[ 1 + \beta \sqrt{\Theta} \exp \left( \frac{\Psi_c (1 - \Theta)}{\Theta} \right) \right] \left[ 1 - \frac{2}{\sqrt{\pi}} \sqrt{-\frac{\Psi_p - \Psi_C}{\sigma}} \exp \left( \sqrt{-\frac{\Psi_p - \Psi_C}{\sigma}} \right) \text{Erfc} \left( \sqrt{-\frac{\Psi_p - \Psi_C}{\sigma}} \right) \right].$$

(22)
The equation (22) is fulfilled only when the electron emission from the collector is critical, i.e. when the electric field at the collector is zero. The equations (20), (21) and (22) form a system of 3 equations for the following unknown quantities: the floating potential $\Psi_C$ of the collector, the value of the plasma potential $\Psi_P$ at the inflection point of the plasma potential and both critical emission coefficients $\gamma_i$ and $\gamma_e$. All the other parameters ($\beta$, $\Theta$, $\tau$, $\mu$ and $\sigma$) are given or selected. Because we have 4 unknown quantities and only 3 equations, we either need one additional equation or we can select one of the emission coefficients $\gamma$ or $\gamma_i$. Let us assume that the ratio $Q$ between the emission coefficients is given in the form:

$$\gamma_i = Q \gamma_e.$$  

(23)

Here $Q$ is a given constant which can have any value between 0 and infinity. The equations (20) – (23) form a system of 4 equations with 4 unknown quantities.

3 RESULTS

In this section we present some results of the model, presented in the previous section. First (Fig. 1) we study the dependence of the solutions of the system (20) – (23) on the assumed ratio $Q$ between the critical emission coefficients. We select the following parameters: $\mu = 1/1836$ (proton mass), $\beta = 0.03$, $\Theta = 20$, $\tau = 1$ and $\sigma = 0.01$. The ratio $Q$ is gradually increased from 0 to 200 in steps of 1. Every time the system (20) – (23) is solved for both potentials $\Psi_P$ and $\Psi_C$ and both critical emission coefficients $\gamma_i$ and $\gamma_e$. It turns out that the potentials $\Psi_P$ and $\Psi_C$ do not depend on $Q$. Their values are: $\Psi_C = -1.14571$ and $\Psi_P = -0.429195$.

Figure 1: Dependence of critical emission coefficients $\gamma_i$ and $\gamma_e$ on $Q$. The other parameters are: $\mu = 1/1836$ (proton mass), $\beta = 0.03$, $\Theta = 20$, $\tau = 1$ and $\sigma = 0.01$. 

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When \( Q \) is gradually increased, \( \gamma_{ec} \) goes to zero and \( \gamma_{ic} \) goes to the limiting value that is reached, when we put \( \gamma = 0 \). Because the potentials \( \Psi_P \) and \( \Psi_C \) do not depend on \( Q \), we can always put one of the emission coefficients to zero and solve only the system of 3 equations (20) - (22) for both potentials and the other emission coefficient. In Fig. 2 we show the dependence of \( \gamma_{ec} \), \( \gamma_{ic} \), \( \Psi_P \) and \( \Psi_C \) on the hot electron density \( \beta \). The following parameters are selected: \( \mu = 1/1836 \), \( \Theta = 20 \), \( \tau = 1 \) and \( \sigma = 0.01 \). The hot electron density \( \beta \) is gradually increased from 0 to 1 in steps of 0.01. Every time the system of equations (20) - (22) is solved two times. First we select \( \gamma = 0 \) and solve the equations (20) - (22) for \( \gamma_{ic} \), \( \Psi_P \) and \( \Psi_C \). Then we put \( \gamma = 0 \) and solve the equations (20) - (22) for \( \gamma_{ec} \), \( \Psi_P \) and \( \Psi_C \). It turns out that both critical emission coefficients when regarded as function of \( \beta \), have very well pronounced maxima. Another observation is that for certain values of \( \beta \) the system of equations (20) - (22) has 3 solutions with 3 different plasma potentials \( \Psi_P \).

![Figure 2: Dependence of critical emission coefficients \( \gamma_{ec} \) and \( \gamma_{ic} \) and potentials \( \Psi_P \) and \( \Psi_C \) on \( \beta \). The other parameters are: \( \mu = 1/1836 \), \( \Theta = 20 \), \( \tau = 1 \) and \( \sigma = 0.01 \).](image)

This means that in certain cases plasmas with different plasma potentials can exist simultaneously in the bounded plasma system. If this is the case, plasmas with different plasma potentials must be separated spatially by a localised potential structure - the double layer.

4 CONCLUSIONS

A modification of a kinetic model of a bounded plasma system has been presented in this paper. The model itself has been developed recently [1]. In this work the model is modified by the introduction of electron and ion emission coefficients. A somewhat surprising result has been obtained that the moments of the particle distribution functions do
not depend on the assumed ratio between the critical ion and the critical electron emission coefficient. The critical emission coefficient for ions is for more than an order of magnitude larger than the corresponding critical emission coefficient for electrons. Both critical emission coefficients, when plotted as functions of $\beta$, have very well pronounced maximum values. Dependence of these maximum values, when other parameters are varied is a subject for further investigation.

REFERENCES