Multiply Scattering of Internal Conversion Electrons in Nuclear Emulsion Used on Beta-Spectrographs

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ABSTRACT

The multiple scattering of β-electrons in nuclear photographic emulsion of β-spectrometer is considered. A numerical calculation method was developed on the basis of many-hit model of emulsion response in frame of the theory of multiple scattering of electrons. The dependences of spectra sensitivity, of ratio signal-noise and other parameters on β-electron energy and thickness of emulsion layer were studied.

1 INTRODUCTION

Electrons of internal conversion of spectra line fall down along normal to the surface of nuclear emulsions used in β-spectrographs [1]. So that after development of irradiated emulsion its optical density $D_S$ can be considered as a signal containing information about structure of radioactive nuclear to be studied. Electrons passing through the emulsion undergo to strong multiple scattering at emulsion atoms. This scattering forms some differential distribution of the density over emulsion depth $Z$ so that each depth gives own yield into final density $D_S$. Besides this distribution there can be a spatial distribution created by some irradiation background, such as γ-irradiation or background electrons and so on. The background irradiation increases optical density by some value $D_b$, which can be taken as measure of a noise. Thus a ratio signal-noise can be written in kind:

$$I = \frac{D_S}{D_b}$$  \hspace{1cm} (1)

2 LIMIT THICKNESS OF EMULSION LAYER.

Obviously the ratio signal-noise depends on emulsion layer thickness. If thickness of photographic layer is greater than residual range of electrons with initial energy $E$, depths with $Z>R(E)$ are unreachable for these electrons (figure 1) and useful yield into summary optical density from this region of depths is zero. The region with small yield into $D_S$ begins practically earlier from some limit depth $L_{lim}(E)$, which is less than residual ranges of electrons $R(E)$. It has place because of distortion of electron trajectories by the multiply scattering. At the same time if there presents background components of radiation with high penetration ability, the yield in $D_b$ may be brought by deeper depths $Z>L_{lim}(E)$ or even
Z=R(E) if the emulsion thickness L greater residual ranges of electrons. Thereby with increasing thickness of emulsion layer the value $D_S(Z)$ after depth $Z$ will be constant meanwhile the ratio signal-noise Eq. (1) will be decreasing. Consequently, choosing the layer thickness equal $L=L_{\text{lim}}(E)$ one can obtain maximal ratio signal-noise with minimal losses in optical density $D_S(L)$.

![Figure 1: Multiply scattering in photographic emulsion layer of electron flux entering into emulsion perpendicular to its surface](image)

### 3 CALCULATION METHOD

In order to find the ration signal-noise we shall calculate the probability of grain development at depth $Z$. We shall use calculation method applied in a work [2] in frame of many-hit reponse and generalized in work [3] for all class of solid state detectors.

Relativeto the emulsion, which we are here considering as one-hit and one-target detector, this method gives relation of photographic action of multiply scattered electrons with photographic action of separate electron. Working region of optical density values for emulsion type-R with 50 μm thickness corresponds to low flux densities of electrons when the probability of passing through the same microcrystal AgBr or passing several electrons through the same crystal is negligibly small. That is why according to work [3] for plane source of electrons for any microscopic crystal AgBr at depth $Z$ the probability of its development can be described by expression:

$$P(Z) = \pi \cdot a_0^2 \cdot N_e \iiint_{\Omega \Omega} P_1^*(s) \cdot f(\Omega,x,y,z,s) d\Omega dx dy ds$$

This equation is good for electron flows with not very high intensity $N_e$, when $\pi a_0^2 N_e << 1$, in opposite case it should be written as

$$P(Z) = 1 - e^{-\pi a_0^2 N_e \iiint_{\Omega \Omega} P_1^*(s) \cdot f(\Omega,x,y,z,s) d\Omega dx dy ds}$$

Where $N_e$ is an amount of electrons emitted by unit surface of a plain electron source, $\tilde{\Omega}$ is a vector in direction of electron moving through point $(x,y,z)$ with residual range $s$. $f(\tilde{\Omega},x,y,z,s)$ is a function of electron distribution density in phase space $(\tilde{\Omega},\tilde{r},s)$. $a_0$ is a radius of microscopic crystal AgBr before emulsion development. $P_1^*(s)$ is the probability this crystal gains ability to developing after passing through it single electron with residual range $s$. 

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In order not to fix number of electrons it is more comfortable to use equation (2) as all calculation can be made per one electron. In opposite case of electron flows with higher intensity \( N_e \) all our results should be recalculated with Eq. (3) for concrete values of \( N_e \).

Though in reality scattering matter occupies half-space \( Z>0 \), we assume that our emulsion is spreading infinitely in both sides from plain electron source and that electrons are emitting in positive direction of direction \( Z \). Influence of magnetic field of \( \beta \)-spectrograph for processes of electron multiply scattering can be ignored as in our case a radius of electron trajectory curvature caused by magnetic field is much greater than total ranges of electrons. That is why multiply electron scattering can be described by the next kinetic equation;

\[
\begin{align*}
- \frac{\partial f}{\partial s} + \cos \theta \frac{\partial f}{\partial z} &= N \cdot \int \sigma(T, \theta^\prime - \theta) \left\{ f(r, \theta^\prime, z) - f(r, \theta, z) \right\} d\theta^\prime + \\
&+ \frac{1}{2 \pi} \delta(z) \cdot \delta(R_o - s) \cdot \delta(\cos \theta - 1),
\end{align*}
\]

Here \( N \) is a number of atoms in one cubic centimeter of emulsion. \( T=T(s) \) is kinetic energy of electron in units of electron rest energy \( m_o c^2 \) with residual range \( s \).

\( \sigma(T, \theta) \) is scattering cross section of electron at separate atom, according to work [5] :

\[
\sigma(T, \theta) = \frac{3}{4} Z(Z+1)^{1/2} \frac{(T+1)^{1/2}}{T^{1/2}(T+1)^{1/2}} \left( 1 + \left( 1 - \cos \theta \right)^2 + \frac{\pi \alpha \beta}{\sqrt{2}} \left( 1 - \cos \theta \right)^{3/2} - \frac{1}{2} \left( \beta^2 + \pi \alpha \beta \right) \left( 1 - \cos \theta \right) \right)
\]

Where \( \eta = \frac{1}{4} \left( \frac{137 \cdot 0.885}{Z^{1/2}} \right)^2 \cdot \frac{1.13 + 3.76 \cdot \alpha^2 (T+1)^2}{T(T+2)} \), \( Z \) is the atomic number of scattering matter and \( \beta \) is an electron velocity in light velocity unit \( c \).

Relation energy-range can be found from the stopping power:

\[
\frac{dE}{ds} = N \sigma_0 \left[ \ln \frac{m_o c^2 T(T+1) \beta^2}{2u^2} + \frac{(1-\beta^2)(-2\sqrt{1-\beta^2} - 1 + \beta^2) \ln 2 + \frac{1}{8} (1 - \sqrt{1-\beta^2})^2}{8} \right]
\]

Here \( u \) is a mean value of ionization energy of the matter.

The probability of emulsion AgBr development \( P_1^+(E) \) after passing through it one electron with energy \( E \) can be used from work [4]:

\[
P_1^+(E) = 1 - e^{-w_r \frac{\Delta E(E)}{E_r}}
\]

where \( w_r = 0.344 \left( \frac{0.076}{a_0} + \frac{1}{3} \left( \frac{0.078}{a_0} \right)^2 \right) \) is a mean number of effective ionizations in one microscopic crystal with radius \( a_0 \) produced by one electron with energy \( E \) in relativistic minimum ionization. \( \Delta E(E) \) is a mean value of energetic losses deposed in a separate crystal by this electron. \( \Delta E_r \) is a some characteristic value of electron energy losses.
In approximation of continuous small energy losses it is possible to consider unequivocal functions $s=s(E)$ and $E=E(s)$ that allows to change function $P_1^+(E)$ for $P_1^+(s)$:

$$P_1^+(s) = 1 - A_1 s^{1/2} - A_2 s^{3/2} - A_3 s^{5/2}$$  \hspace{1cm} (8)$$

Coefficients $A_1$, $A_2$ and $A_3$ can be found by method of least squares using Eqs.(4), (5) and (6). Knowledge of function $P_1^+(s)$ gives a possibility to find moments of function on depth $Z$ using (2) and (3):

$$\left\langle z^n \right\rangle = \int_{-1}^{+1} z^n P_1^+(z)dz$$  \hspace{1cm} (9)$$

From these moments it is possible to reconstruct the probability of grain development as a function on depth $P_1^+(Z)$. Using reconstruction method of Spencer [5] we can write this probability in kind:

$$P(z) = \frac{1}{2} \left( \sum_{i=1}^{3} A_i^n e^{-a_i^2 \frac{(z-\beta_i)^2}{\beta_i^2}} + \text{sgn}(z) \sum_{i=1}^{3} A_i^n e^{-a_i^2 \frac{(z+\beta_i)^2}{\beta_i^2}} \right)$$  \hspace{1cm} (10)$$

Here $\alpha$, $A_i^n$, $A_i^u$, $\beta_i^n$, $\beta_i^u$ are parameters of Spencer’s method.

**Discussion**

In general case optical density of photographic emulsion $D$ is a function of emulsion thickness $L$ and of electron energy $E$. For not very intense electron irradiation the optical darkness is proportional to the next integral over its depth $Z$:

$$D_3(L,E) \sim Q(L,E) = \int_0^L P_1^+(z,E)dz$$  \hspace{1cm} (11)$$

If we fix emulsion thickness $L$ it will be possible to find sensitivity $\varepsilon(E)$ of emulsion as function on electron energy $E$ for given thickness $L$:

$$\varepsilon(E) = Q(L = \text{const}, E)$$  \hspace{1cm} (12)$$

Figure 2 presents comparison of calculated and experimental [6] spectral sensitivities $\varepsilon(E)$ for nuclear emulsion type-R with $L=50 \mu m$. 
Figure 2: Comparison of theoretical and experimental spectral sensitivity for nuclear emulsion type-R with thickness L=50 μm.

The function ε(E) goes through a maximum as it is seen from figure 2. A graphical view of calculated distribution of developed grains over emulsion depth is shown in figure 3 (Curve 1). Curve 2 presents calculated dependence of integrated amount of developed grains on layer thickness L:

\[ Q(L,E) = \int_0^L P^*(z,E)dz \]  

\[ \text{(13)} \]

Figure 3: Differential (curve 1) and integral distributions (curve 2) over depth of nuclear emulsion type-R.

We can see that the differential distribution of developed grains number over emulsion depth has a maximum. Integral distribution Eq. (11) \( Q(L) \) after some thickness goes at the plateau. This value of thickness we shall call as a limit thickness \( L_{lim}(E) \). Further increasing of the thickness will not give any remarkable growth of \( Q(L) \) or \( D_\beta(L) \).

Now we consider case, when background created by any radiation with strong penetration possibility consists of developed grains homogenously distributed all over the nuclear emulsion volume. Then the dependence of ration signal-noise \( I(Z) \) on emulsion layer thickness \( L \) can be written in the next form:

\[ I(L) = K \frac{Q(L)}{L} \]  

\[ \text{(14)} \]
Here K is a constant value depending on nature of the background. Figure 4 presents functions I(L/R) on the depth which expressed in units of total range of electrons \( R(E) \) for two extreme points of electron energy interval for \( \beta \)-spectrographs: \( E_1=10 \) keV and \( E_2(L/R)=500 \) keV. Scales \( I_1(L/R) \) and \( I_2(L/R) \) in the figure are chosen so that at \( L/R=1 \) both signal-noise ratios are equal unit: \( I(L/R=1)=1 \).

![Figure 4: Dependence of ratio signal-noise on photographic layer thickness L for recording by monoenergetic electron beam when background homogenously distributed over the layer volume. 1 – ratio signal function for electron flux 10 keV; 2 – the same function for 500 keV electron flux.](image)

As it can be seen from the figure 4, the maximum of second ratio \( I_2(L/R) \) for \( E_2=500 \) keV is situated more to the left than \( I_1(L/R) \) for \( E_1=10 \) keV. It can be explained by next facts: with increasing energy E of electron grows its range R, with R grows electrons lose more information about its initial direction of emitting, fraction of backscattered electrons is increasing, maximum of differential distribution \( P(Z) \) moves to the left, as it shows figure 5. The maximum of \( I(L/R) \) moves to the left too.

![Figure 5: Distribution of developed grains over emulsion layer depth after exposures by electron flux with energy E=10 keV - curve 1 and E=500 keV – curve 2.](image)

As it is seen from figure 4 if we agree loose some value in optical density because of reducing thickness of the photographic layer, we can gain in increasing of signal-noise ratio. With this aim it is necessary to use thickness less then the limit thickness \( L < L_{lim}(R) \). But if one uses too thin thickness then the surface of emulsion layer can turned out to be moved to
the left from the maximum of $I(L)$, for example, $L=50 \, \mu m$ for electrons with energy $E=500$ keV. It will be a case when the signal-noise ratio is not maximal, but losses in optical density $D_S$ are great.

This consideration is applicable also for inhomogeneous background irradiations with photographic action depending on depth inside emulsion layer. If the depth with maximal background is known it possible to exclude this maximum at the expanse of lose in useful optical density $D_S$ by taking correspondent thickness $L_{\text{lim}}$. With this aim a set limited thickness $L_{97\%}$, $L_{90\%}$, $L_{80\%}$, $L_{70\%}$, $L_{60\%}$, giving optical densities 97, 90, 80, 70, 60% was calculated. The calculation results are presented in table 1.

Each calculated thickness presents some part $C(E)$ of residual electron range, i.e. $C(E)=L_{\%}(E)/R(E)$, which are given in table 2. Value $C(E)$ decreases with energy increasing, that is in accordance with conclusions deducing from figures 4 and 5.

<table>
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<tr>
<th>E keV:</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
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<th>200</th>
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<tr>
<td>$L_{60%}$</td>
<td>0.41</td>
<td>1.16</td>
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<table>
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<tr>
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Thus, choosing from tables 1 and 2 limit thickness in accordance with conditions of experiment it is possible to obtain highest ratio signal-noise. It is necessary to emphasize that our approach and numerical results are available not only for work with $\beta$-spectrograph, but for any other experiments with photographic emulsion irradiation by electrons at presence any background, for example, any radiation with large penetrating ability.
References


