A One-dimensional Kinetic Model of a Current-voltage Characteristics of an Electron Emitting Electrode that Terminates a Bounded Plasma System Containing a Two-electron Temperature Plasma

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ABSTRACT

A fully kinetic one-dimensional model of potential formation in a bounded plasma system that contains a two-electron temperature plasma and is terminated by an electron emitting electrode (collector) that we have already presented earlier [1] has in this work been expanded to include non-zero drift velocities of all the particle species. A complete current-voltage characteristics of the collector is obtained. It is found that at certain conditions the current voltage characteristics may have 3 different floating potentials.

1 INTRODUCTION

In the edge plasmas of fusion devices energetic electron populations are readily produced. The presence of energetic electrons has a remarkable effect on the potential formation in the plasma and consequently on particle losses to the wall. Also electron emission from the walls that limit the plasma is a common occurrence in fusion devices. In this work we present a fully kinetic model of plasma potential formation in a bounded plasma system containing singly charged positive ions and 3 groups of electrons: the cool the hot and the emitted. The velocity distributions of all 4 particle groups are assumed to be drifted maxwellians.

2 THE MODEL

We consider a large planar electrode (collector) with its surface perpendicular to the x axis of the coordinate system. The collector is located at x = 0. This electrode absorbs all the particles that hit it. On the other hand it may also emit electrons. This electron emission may be thermal or secondary, triggered by the impact of incoming electrons and/or ions. The details of the emission mechanism are not essential for the model.

An infinitely large planar plasma source also has its surface perpendicular to the x axis. The source is located at a certain distance x = L from the collector. The distance L is not
essential for the model and may be taken arbitrarily. The source injects 3 groups of charged particles into the system: singly charged positive ions (index $i$), the cool electrons (index 1), the hot electrons (index 2). The electrons that are emitted from the collector have index 3. The particles $i$, 1 and 2 are injected from the source with half-maxwellian velocity distribution function with respective temperatures $T_i$, $T_1$ and $T_2$. The emitted electrons also have a half-maxwellian velocity distribution function at the collector with the temperature $T_3$. We assume that $T_2 > T_1$ and $T_1 >> T_3$. The ion temperature $T_i$ can in principle be arbitrary, but is usually taken smaller than or equal to $T_1$.

The collector is biased to a certain (negative!) potential $\Phi_C$. The potential and the electric field at the source are set to zero. This imposes the following boundary conditions:

$$\Phi(x = L) = 0, \quad \frac{d\Phi}{dx}(x = L) = 0,$$

for the Poisson equation:

$$\frac{d^2\Phi}{dx^2} = -\frac{e_0}{\varepsilon_0} \left( n_i(x) - n_1(x) - n_2(x) - n_3(x) \right),$$

where $e_0$ is the elementary charge and $n_j(x)$ are the respective particle densities.

An ion that has left the source with a negligibly small initial velocity has at the distance $x < L$ from the collector the velocity:

$$v_{mi}^i = -\sqrt{-\frac{2e_0\Phi(x)}{m_i}},$$

in the direction towards the collector. The distribution function for the ions can therefore be written in the following way:

$$f_i = n_i \sqrt{\frac{m_i}{2\pi kT_i}} \exp \left( -\frac{e_0\Phi(x)}{kT_i} \right) \exp \left( -\frac{m_i (v + v_i)^2}{2kT_i} \right) H \left( -v - \sqrt{-\frac{2e_0\Phi(x)}{m_i}} \right),$$

where $v_i$ is the drift velocity of the ions in the direction towards the collector.

An electron that has left the collector source with a negligibly small initial velocity has at the distance $x$ from the collector the velocity:

$$v_{me}^e = \sqrt{\frac{2e_0(\Phi(x)-\Phi_C)}{m_e}},$$

in the direction towards the source. The distribution functions for the cool and for the hot electrons at the distance $x$ from the collector can therefore be written in the following way:

$$f_1 = n_1 \sqrt{\frac{m_1}{2\pi kT_1}} \exp \left( \frac{e_0\Phi(x)}{kT_1} \right) \exp \left( -\frac{m_1 (v + v_1)^2}{2kT_1} \right) H \left( -v + \sqrt{\frac{2e_0(\Phi(x)-\Phi_C)}{m_e}} \right),$$

$$f_2 = n_2 \sqrt{\frac{m_2}{2\pi kT_2}} \exp \left( \frac{e_0\Phi(x)}{kT_2} \right) \exp \left( -\frac{m_2 (v + v_2)^2}{2kT_2} \right) H \left( -v + \sqrt{\frac{2e_0(\Phi(x)-\Phi_C)}{m_e}} \right).$$

Here $v_1$ and $v_2$ are the corresponding drift velocities of the cool and of the hot electrons in the direction towards the collector. On the other hand the distribution function of the emitted electrons can be written in the following way:

$$f_3 = n_3 \sqrt{\frac{m_e}{2\pi kT_3}} \exp \left( \frac{e_0(\Phi(x)-\Phi_C)}{kT_3} \right) \exp \left( -\frac{m_e (v - v_3)^2}{2kT_3} \right) H \left( v - \sqrt{\frac{2e_0(\Phi(x)-\Phi_C)}{m_e}} \right),$$

where $v_3$ is the drift velocity of the emitted electrons in the direction towards the source. The following variables are introduced:
\[ \mu = \frac{m}{m_i}, \quad \tau = \frac{T}{T_i}, \quad \Theta = \frac{T}{T_i}, \quad \sigma = \frac{T}{T_i}, \quad \Psi = \frac{e\Phi(x)}{kT_i}, \quad \Psi_c = \frac{e\Phi(x) = 0}{kT_i}, \quad \alpha = \frac{n}{n_i}, \quad \beta = \frac{n_0}{n_i}. \] 

(9)

\[ \varepsilon = \frac{n_0}{n_i}, \quad v_0 = \sqrt{\frac{2kT_i}{m}}, \quad z = \frac{x}{\lambda_D}, \quad \lambda_D = \sqrt{\frac{m}{n_0 e^2}}, \quad u = \frac{v}{v_0}, \quad u_i = \frac{v_i}{v_0}, \quad u_0 = \frac{v_0}{v_0}, \quad u_3 = \frac{v_3}{v_0}. \]

With these variables the distribution functions are written in the following way:

\[ F_i(u, \Psi) = \frac{\alpha}{\sqrt{\pi} \tau \mu} \exp\left(-\frac{\Psi}{\tau}\right) \exp\left(-\frac{(u+u_i)^2}{\mu \tau}\right) H\left(-u - \sqrt{\mu \Psi}\right), \]

\[ F_1(u, \Psi) = \frac{1}{\sqrt{\pi}} \exp\left(\Psi\right) \exp\left(-\left(u+u_i\right)^2\right) H\left(-u + \sqrt{\Psi - \Psi_c}\right), \]

\[ F_2(u, \Psi) = \frac{\beta}{\sqrt{\pi} \Theta} \exp\left(\frac{\Psi}{\Theta}\right) \exp\left(-\frac{(u+u_3)^2}{\Theta}\right) H\left(-u + \sqrt{\Psi - \Psi_c}\right), \]

\[ F_3(u, \Psi) = \frac{\varepsilon}{\sqrt{\pi} \sigma} \exp\left(\frac{\Psi - \Psi_c}{\sigma}\right) \exp\left(-\frac{(u-u_3)^2}{\sigma}\right) H\left(u - \sqrt{\Psi - \Psi_c}\right). \]

(10)

The zero moments of the distribution functions give the respective particle densities, while the first moments give the fluxes:

\[ N_j(\Psi) = \int F_j(u, \Psi) du, \quad J_j(\Psi) = \int u F_j(u, \Psi) du. \] 

(11)

Here \( j \) stands for all the particle species \( j = i, 1, 2 \) and \( 3 \). When the densities \( N_j \) are found from the first of the equations (11) and inserted into the Poisson equation (2) the following equation is obtained:

\[ \frac{d^2 \Psi}{dz^2} = -\frac{\alpha}{2} \exp\left(-\frac{\Psi(z)}{\tau}\right) \text{Erfc}\left(\frac{\sqrt{\mu} \Psi(z) - u}{\mu \tau}\right) + \frac{1}{2} \exp(\Psi(z)) \left[1 + \text{Erf}\left(u_i + \sqrt{\Psi(z) - \Psi_c}\right)\right] + \frac{\beta}{2} \exp\left(\frac{\Psi(z)}{\Theta}\right) \left[1 + \text{Erf}\left(u_3 + \sqrt{\Psi(z) - \Psi_c}\right)\right] + \frac{\varepsilon}{2} \exp\left(\frac{\Psi(z) - \Psi_c}{\Theta}\right) \text{Erfc}\left(\frac{\sqrt{\Psi(z) - \Psi_c} - u_3}{\Theta}\right) \]

(12)

where:

\[ \text{Erf}(x) = \frac{2}{\sqrt{\pi}} \int_{0}^{x} \exp(-t^2) dt, \quad \text{Erfc}(x) = \frac{2}{\sqrt{\pi}} \int_{x}^{\infty} \exp(-t^2) dt. \]

(13)

At a certain distance \( z = z_0 \) from the collector the plasma is neutral and the potential at that point has the value \( \Psi(z = z_0) = \Psi_p \). So the neutrality condition can be obtained directly from (12):

\[ 0 = -\alpha \exp\left(-\frac{\Psi_p}{\tau}\right) \text{Erfc}\left(\frac{\sqrt{\mu} \Psi_p - u_i}{\mu \tau}\right) + \exp(\Psi_p) \left[1 + \text{Erf}\left(u_i + \sqrt{\Psi_p - \Psi_c}\right)\right] + \frac{\beta}{2} \exp\left(\frac{\Psi_p}{\Theta}\right) \left[1 + \text{Erf}\left(u_3 + \sqrt{\Psi_p - \Psi_c}\right)\right] + \frac{\varepsilon}{2} \exp\left(\frac{\Psi_p - \Psi_c}{\Theta}\right) \text{Erfc}\left(\frac{\sqrt{\Psi_p - \Psi_c} - u_3}{\Theta}\right). \]

(14)

At \( z = z_0 \) the potential \( \Psi(z) \) obviously has an inflection point. Since:

\[ \frac{1}{2} \frac{d}{dz} \left(\frac{d\Psi}{dz}\right)^2 = \frac{d^2 \Psi}{dz^2}, \]

(15)

the Poisson equation (12) can be multiplied by \( d\Psi/dz \) and integrated once over \( \Psi \) from \( \Psi = 0 \) (at the source) to \( \Psi = \Psi_p \) (at the inflection point). The following equation is obtained:
This equation is called the zero electric field condition at the inflection point. If the electron emission from the collector increases, at a certain level of emission a negative space charge may start to accumulate at the collector surface. At certain critical electron emission the electric field that accelerates the negative electrons away from the collector becomes equal to zero. The zero electric field condition at the collector can be derived in a very similar way as the zero electric field condition at the inflection point (16). Again the Poisson equation (12) is multiplied by $d\Psi/dz$ and integrated once over $\Psi$, only now the integration limits go from $\Psi = \Psi_p$ (at the inflection point) to $\Psi = \Psi_C$ (at the collector). The following equation is obtained:
\[ \left( \frac{d\Psi}{dz} \right)_{\Psi=\Psi_c} = -\alpha \tau \left[ \frac{1}{u_i} \sqrt{\frac{\mu \tau}{\pi}} \exp\left( -\frac{\Psi_{r}^2}{\mu \tau} \right) \exp\left( \frac{2u_i \sqrt{\frac{\Psi_{r}}{\tau}}}{\mu \tau} \right) - \exp\left( \frac{2u_i \Psi_{r}}{\tau} \sqrt{\frac{\Psi_{c}}{\tau}} \right) \right] + \exp\left( \Psi_{c} \right) \left[ 1 + \text{Erfc}(u_i) + \frac{1}{u_i \sqrt{\pi}} \exp\left( -u_i^2 \right) \right] \left[ 1 + \exp\left( -2u_i \sqrt{\frac{\Psi_{r} - \Psi_{c}}{\tau}} \right) \right] + \exp\left( \Psi_{p} \right) \left[ 1 + \text{Erfc}(u_i + \sqrt{\Psi_{r} - \Psi_{c}}) \right] \]

\[ + \beta \Theta \left[ \frac{\sqrt{\sigma}}{u_i \sqrt{\pi}} \exp\left( -\frac{u_i^2}{\sigma} \right) \left( 1 - \exp\left( \frac{2u_i \sqrt{\Psi_{r} - \Psi_{c}}}{\sigma} \right) \right) + \text{Erfc}\left( -\frac{u_i \sqrt{\sigma}}{\Psi_{r}} \right) \right] \]

\[ + \epsilon \Theta \left[ -\exp\left( \Psi_{p} \right) \text{Erfc}\left( \frac{\sqrt{\Psi_{r} - \Psi_{c}} - u_i}{\sqrt{\sigma}} \right) \right] = 0. \]

The total current density \( J_t \) to the collector is the sum of the contributions of all 4 groups of charged particles. We take the ions and the emitted electrons with a positive sign, while the cool and the hot electrons are taken with the negative sign:

\[ J_t = \frac{1}{2 \sqrt{\pi}} \alpha \exp\left( -\frac{\Psi_{r} \Psi_{c} - u_i^2}{\tau} \right) + \beta \Theta \exp\left( \frac{\Psi_{r} - \Psi_{c}}{\Theta} \right) + \epsilon \Theta \left( -\exp\left( \frac{\Psi_{r} \Psi_{c} - u_i^2}{\tau} \right) + \text{Erfc}\left( \frac{\Psi_{r} \Psi_{c} - u_i^2}{\sigma} \right) \right) \]

Here a remark about the drift velocities should be added. If any of the drift velocities \( u_i, u_1, u_2 \) and \( u_3 \) is zero, then the Poisson equation (12), the neutrality condition (14) and the total current density (18) keep their form only the zero value of the corresponding drift velocity is inserted into the equations. Both zero electric field conditions are obtained by the integration of the densities (zero moments of the distribution functions) over the potential and because of the drift velocities enter into the denominators. This causes a singularity if any of the drift velocities is set to zero. The correct way to derive both zero electric field conditions in the case of zero drift velocities is the following. First the densities must be obtained from the first
of the equations (11) with zero drift velocity already inserted into the corresponding velocity distribution function and then the density obtained in this way must be integrated over the potential. When all the drift velocities are zero, the zero electric field condition at the inflection point reads:

\[
\left( \frac{d\Psi}{dz} \right)_{\Psi=\Psi_C}^2 = -\alpha \tau \left[ 1 - \frac{2}{\sqrt{\pi}} \sqrt{\frac{\Psi_p}{\tau}} - \exp \left( -\frac{\Psi_p}{\tau} \right) \text{erfc} \left( \frac{\Psi_p}{\tau} \right) \right] + \exp (\Psi_P) \left[ 1 + \text{erf} \left( \sqrt{\frac{\Psi_p-\Psi_C}{\Theta}} \right) - \frac{2}{\sqrt{\pi}} \exp (\Psi_C) \left( \sqrt{\frac{\Psi_p-\Psi_C}{\Theta}} - \sqrt{\frac{-\Psi_C}{\Theta}} \right) \right] + \beta \Theta \left[ \exp \left( \frac{\Psi_C}{\Theta} \right) \left( 1 + \text{erf} \left( \sqrt{\frac{\Psi_p-\Psi_C}{\Theta}} \right) - \frac{2}{\sqrt{\pi}} \exp \left( \frac{\Psi_C}{\Theta} \right) \left( \sqrt{\frac{\Psi_p-\Psi_C}{\Theta}} - \sqrt{\frac{-\Psi_C}{\Theta}} \right) \right) \right] + \epsilon \sigma \left[ 2 \left( \sqrt{\frac{\Psi_p-\Psi_C}{\sigma}} - \sqrt{\frac{-\Psi_C}{\sigma}} \right) + \exp \left( \frac{\Psi_p-\Psi_C}{\sigma} \right) \right] \text{erfc} \left( \frac{\Psi_p-\Psi_C}{\sigma} \right) - \exp \left( \frac{-\Psi_C}{\sigma} \right) \text{erfc} \left( \frac{-\Psi_C}{\sigma} \right) = 0,
\]

(19)

while the zero electric field condition at the collector reads:

\[
\left( \frac{d\Psi}{dz} \right)_{\Psi=\Psi_C}^2 = -\alpha \tau \left[ \exp \left( \frac{-\Psi_p}{\tau} \right) \text{erfc} \left( \sqrt{\frac{-\Psi_C}{\tau}} \right) - \exp \left( -\frac{-\Psi_C}{\tau} \right) \text{erfc} \left( \sqrt{\frac{-\Psi_C}{\tau}} \right) + \frac{2}{\sqrt{\pi}} \left( \sqrt{\frac{-\Psi_p}{\tau}} - \sqrt{\frac{-\Psi_C}{\tau}} \right) \right] + \exp (\Psi_C) \left[ 1 + \text{erf} \left( \sqrt{\frac{-\Psi_p-\Psi_C}{\Theta}} \right) \right] + \beta \Theta \left[ \exp \left( \frac{-\Psi_C}{\Theta} \right) \left( 1 + \text{erf} \left( \sqrt{\frac{-\Psi_p-\Psi_C}{\Theta}} \right) \right) - \exp \left( \frac{-\Psi_C}{\Theta} \right) \left( \sqrt{\frac{-\Psi_p-\Psi_C}{\Theta}} - \sqrt{\frac{-\Psi_C}{\Theta}} \right) \right] + \epsilon \sigma \left[ 1 - \frac{2}{\sqrt{\pi}} \left( \sqrt{\frac{-\Psi_p-\Psi_C}{\sigma}} - \sqrt{\frac{-\Psi_C}{\sigma}} \right) \right] \text{erfc} \left( \frac{-\Psi_p-\Psi_C}{\sigma} \right) - \exp \left( \frac{-\Psi_C}{\sigma} \right) \text{erfc} \left( \frac{-\Psi_C}{\sigma} \right) = 0.
\]

(20)

3 RESULTS

We select the parameters \( \mu = 1/1836, \tau = 0.1, \Theta = 65, \sigma = 0.08, u_2 = 1.6, u_3 = 0.021 \) and \( u_i = u_1 = 0 \) while \( \beta \) is gradually increased. For each set of the parameters the system of equations (14), (16), (17) and (18) with \( J_i = 0 \) is solved for \( \alpha, \epsilon, \Psi_P \) and \( \Psi_C \). Because 2 of the drift velocities are zero the zero electric field conditions (16) and (17) must be combined appropriately with (19) and (20). The result is shown in figure 1. For the values of \( \beta \) around 0.17 the system of equations (14), (16), (17) and (18) may have up to 5 solutions. This means that for a given set of the parameters of the plasma production the collector may have up to 5 different floating potentials. This is additionally illustrated in figure 2, where the current voltage (dependence of \( J_t \) on \( \Psi_C \)) is shown for the parameters: \( \mu = 1/1836, \tau = 0.1, \Theta = 65, \sigma = 0.08, \beta = 0.17, u_2 = 1.6, u_3 = 0.021 \) and \( u_i = u_1 = 0 \). For every selected \( \Psi_C \) the system of equations (14), (16) and (17) is solved for \( \alpha, \epsilon, \Psi_P \) and then \( J_t \) is calculated using (18). Again the zero electric field conditions (16) and (17) must be combined appropriately with (19) and (20) This means that the electron emission current is space charge limited all the time. In figure 3 we show the current voltage characteristics for \( \tau = 0.1, \Theta = 65, \sigma = 0.08, \beta = 0.17 \) and \( u_i = u_1 = 0 \), while the drifts \( u_2 \) and \( u_3 \) are varied. It is assumed that the electron emission current is space charge limited all the time, so for every set of the parameters the system of equations (14), (16) and (17) is solved for \( \alpha, \epsilon, \Psi_P \) and then \( J_t \) is calculated using (18).
Figure 1: The solutions $\alpha$, $\varepsilon$, $\Psi_P$ and $\Psi_C$ of the system of equations (14), (16), (17) and (18) with $J_i = 0$ versus $\beta$ for the parameters $\mu = 1/1836$, $\tau = 0.1$, $\Theta = 65$, $\sigma = 0.08$, $u_2 = 1.6$, $u_3 = 0.021$ and $u_i = u_1 = 0$.

Figure 2: The current voltage characteristics and the solutions $\alpha$, $\varepsilon$ and $\Psi_P$ for the parameters $\mu = 1/1836$, $\tau = 0.1$, $\Theta = 65$, $\sigma = 0.08$, $u_2 = 1.6$, $u_3 = 0.021$, $u_i = u_1 = 0$ and $\beta = 0.17$. 
Figure 3: The current voltage characteristics for various values of the drifts $u_2$ and $u_3$. The other parameters are: $\mu = 1/1836$, $\tau = 0.1$, $\Theta = 65$, $\sigma = 0.08$, $\beta = 0.17$ and $u_i = u_1 = 0$.

4 CONCLUSIONS

We have expanded a one-dimensional kinetic model of a bounded plasma system which has been developed earlier [1] to include the non-zero drift velocities in the distribution functions of all the particle species. In the presence of the hot electrons in the plasma and under the space charge limited emission of the electrons from the collector the model predicts a triple floating potential of the collector. Such triple crossings of the zero current line have been observed in experiments [2,3]. Drifts in the distribution functions of the hot and of the emitted electrons have a strong impact to the current voltage characteristics of the collector.

ACKNOWLEDGMENTS

This work has been carried out within the Association EURATOM-MHST. The content of the publication is the sole responsibility of its authors and it does not necessarily represent the view of the Commission or its services.

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