ABSTRACT

Modern Monte Carlo transport codes in combination of fast computer clusters enable very accurate calculations of the most important reactor kinetic parameters. Such are the effective delayed neutron fraction, $\beta_{\text{eff}}$, and mean neutron generation time, $\Lambda$. We calculated the $\beta_{\text{eff}}$ and $\Lambda$ for various realistic and hypothetical annular TRIGA Mark II cores with different types and amount of fuel. It can be observed that the effective delayed neutron fraction strongly depends on the number of fuel elements in the core or on the core size. E.g., for 12 wt. % uranium standard fuel with 20 % enrichment, $\beta_{\text{eff}}$ varies from 0.0080 for a small core (43 fuel rods) to 0.0075 for a full core (90 fuel rods). It is interesting to note that calculated value of $\beta_{\text{eff}}$ strongly depends also on the delayed neutron nuclear data set used in calculations. The prompt neutron life-time mainly depends on the amount (due to either content or enrichment) of $^{235}\text{U}$ in the fuel as it is approximately inversely proportional to the average absorption cross-section of the fuel. E.g., it varies from 28 $\mu$s for 30 wt. % uranium content fuelled core to 48 $\mu$s for 8.5 wt. % uranium content LEU fuelled core. Description of the calculation method and detailed results are presented in the paper.

1 INTRODUCTION

The most important parameters in reactor kinetics are the effective delayed neutron fraction $\beta_{\text{eff}}$ and mean neutron generation time, $\Lambda$. In research reactors (e.g. TRIGA) these parameters are usually provided by the manufacturer in the design phase and are not calculated for various core conditions. As the reactor kinetic parameters strongly depend on the fuel type and core configuration, it is very important to take this into account especially when performing core conversion.

It is very difficult to measure $\beta_{\text{eff}}$ and $\Lambda$ separately (only their ratio, $\beta_{\text{eff}}/\Lambda$, can easily be measured), hence these parameters are usually determined only by calculation. In 1965 Keepin provided the theoretical foundations for such calculations [5], however, related to deterministic transport equation. His approach requires calculation of flux and its adjoint function as a weighting function. The method is very difficult to apply as it requires detailed multigroup transport calculations. As well, it can not be applied in continuous-energy Monte Carlo codes.

The introduction of delayed neutrons into MCNP, starting from version 4C [1],[15] enabled the development of Monte Carlo estimators for $\beta_{\text{eff}}$ based on other methods. Several methods have been proposed for calculation of $\beta_{\text{eff}}$ with Monte Carlo method. They are thoroughly described in the papers by Nauchi and Kameyama [9], Spriggs et al.[12] and by Meulekamp and van der Merck [7]. Most of these methods require modifying the MCNP
source code. Only the so called prompt method [7] can be used without modifications of standard MCNP code. It has been shown in paper by Meulekamp and van der Merck that the prompt and the "Meulekamp-Merck" method give similarly good results of $\beta_{\text{eff}}$ for a set of various benchmarks. However, the prompt method is much more time consuming, as it requires about 40 times longer runs. Nevertheless this is not an important issue as the typical MNCP input computation time is decreasing rapidly (it is halved almost every year) with the development of faster computer hardware. The $k$-ratio method [12] also yields good results except for heterogeneous systems [7]. It has also been shown that the differences in calculated values of $\beta_{\text{eff}}$ with different data sets (JEFF 3.0, ENDF/B-VI, JENDL-3.3) are sometimes larger than the difference between the three methods.

The purpose of this paper is to systematically calculate $\beta_{\text{eff}}$ and $\Lambda$ for various realistic and hypothetical annular 250 kW TRIGA Mark II cores with different types and amount of fuel using MCNP code. Four types of fuel rods (FR) are considered: 8.5 FR, 12 FR, 20 FR and 30 FR, each containing 8.5 w/o, 12 w/o, 20 w/o and 30 w/o of uranium, respectively. All of the fuel rods are 20 % enriched and fresh (zero burn-up). In addition, the differences between the calculated values of $\beta_{\text{eff}}$ and $\Lambda$ due to different cross section libraries are investigated.

2 CALCULATION METHOD AND MODEL

MCNP [1] computer code was used in the calculations. MCNP is a general-purpose, continuous-energy, generalized-geometry Monte Carlo transport code. The calculations reported in this paper were performed with version 5.1.40 of the code.

In MCNP it is possible to model the reactor in 3-D geometry in details. Simplifications of the geometry were done only by simplifying surroundings of the core to an extent which does not affect $k_{\text{eff}}$ significantly [3]. The fuel rod was modelled exactly, meaning that Zr rod, stainless steel cladding, air gaps and Mo supporting disc were modelled explicitly. The supporting grid, graphite reflector with rotary groove and central irradiation channel in the core were also explicitly modelled.

The reactor core model was very similar to the one used for benchmark evaluation of TRIGA Mark II reactor [3], the main difference being in control rods and number of fuel rods. Zero power and no-xenon conditions were assumed. In order to observe $\beta_{\text{eff}}$ and $\Lambda$ resulting purely from differences in fuel composition and core size and not from other perturbations (e.g. empty positions, irradiation channels, etc.) the core was made as uniform as possible by replacing the empty positions and irradiation channels with fuel rods. All fuel rods in the core were considered to be fresh. Schematic view of the benchmark TRIGA core is shown in Figure 3.

Three different nuclear data libraries were used in our calculations; ENDF/B-VI.8, ENDF/B-VII [2] and JEFF 3.1 [8].
2.1 Calculation of $\beta_{\text{eff}}$

We used the so called prompt method to calculate $\beta_{\text{eff}}$. Effective delayed neutron fraction, $\beta_{\text{eff}}$, was calculated in the following way. First we calculated the multiplication factor by considering both the prompt and delayed neutrons, denoted $k$. In the second calculation we used only prompt neutrons to calculate the multiplication factor, denoted $k_p$. The MCNP calculations were run with 100,000,000 active histories. A total of 100,000 histories per generation were used and 1,010 generations of neutrons. The first 10 generations were skipped to obtain a well-distributed neutron source. The multiplication factor had statistical error (one standard deviation) from 0.00006 to 0.00008. The $\beta_{\text{eff}}$ was then calculated by using the following formula [7].

\[
\beta_{\text{eff}} = 1 - \frac{k_p}{k}
\]  

(1)

$\beta_{\text{eff}}$ was calculated for four critical TRIGA cores, each containing different type of fuel elements. The calculated values of $\beta_{\text{eff}}$ for various realistic and hypothetical TRIGA mark II cores containing different types of fuel elements are presented in Table 2.

2.2 Calculation of $\Lambda$

The mean neutron generation time, $\Lambda$, was calculated by using the pulsed neutron technique [11]. This method is commonly used for reactivity measurements; a burst of neutrons is injected into a subcritical system and the decay of neutron population is observed as a function of time [5]. After neutron thermalization and decay of higher flux modes, the fundamental-mode decay constant $\alpha_0$ can be measured. The relation between $\alpha_0$ and reactivity, $\rho$, in a multiplying system without sources is obtained from the basic definition of decay constant (i.e. relative change of neutron density n per unit time).
\[
\alpha_0 = \frac{1}{n} \frac{dn}{dt} = \frac{\rho - \beta_{\text{eff}}}{\Lambda}
\]  \tag{3}

where \( \Lambda \) is the mean neutron generation time, related to prompt neutron lifetime by \( \Lambda = \frac{l}{k} \).

If \( \rho \) and \( \beta_{\text{eff}} \) are known, this method can be used for mean neutron generation time (or prompt neutron lifetime) determination.

MCNP computer code was used to simulate the neutron source and prompt neutron decay. A neutron source with fission spectrum was placed at the centre of the slightly subcritical reactor core. Three million source neutrons were generated within 35 ms. The source was active 35 ms in order to obtain the fundamental mode of neutron flux distribution. The neutron flux was tallied over every fuel element as a function of time. After 35 ms the source was turned off and the prompt drop was calculated (Figure 2). The decay constant was obtained by fitting the prompt drop with the first order exponential decay function, \( n(t) = n_0 e^{-\alpha t} + N \), where \( n \) denotes the average number of neutrons tallied in all fuel elements and \( N \) denotes a constant. The observed value of \( \alpha_0 \) was found to be independent of the fuel element in which the neutron flux was tallied, indicating the adequacy of a space-independent kinetics analysis. The mean neutron generation time was calculated using the following formula

\[
\Lambda = \frac{\rho - \beta_{\text{eff}}}{\alpha_0}
\]  \tag{4}

The multiplication factor (or reactivity) of the system was calculated in a separate run. \( \beta_{\text{eff}} \) was calculated by the prompt method described in section 2.1. The mean neutron generation times were calculated for various realistic and hypothetical TRIGA mark II cores containing different types of fuel elements. The results are presented (together with \( \beta_{\text{eff}} \) values) in Table 1.

Table 1: Mean neutron generation time and effective delayed neutron fraction for various TRIGA cores calculated with MCNP, using ENDF/B-VII nuclear data.

<table>
<thead>
<tr>
<th>Fuel type</th>
<th>no. of FR</th>
<th>( k_{\text{eff}} )</th>
<th>( \beta_{\text{eff}} ) (pcm)</th>
<th>( \Lambda ) (( \mu )s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.5 FR</td>
<td>55</td>
<td>0.99825 ± 0.00006</td>
<td>740 ± 10</td>
<td>47.7 ± 0.2</td>
</tr>
<tr>
<td>12 FR</td>
<td>41</td>
<td>0.99465 ± 0.00006</td>
<td>751 ± 10</td>
<td>42.0 ± 0.3</td>
</tr>
<tr>
<td>20 FR</td>
<td>42</td>
<td>0.99847 ± 0.00006</td>
<td>730 ± 10</td>
<td>31.9 ± 0.3</td>
</tr>
<tr>
<td>30 FR</td>
<td>43</td>
<td>0.99695 ± 0.00006</td>
<td>758 ± 11</td>
<td>28.1 ± 0.4</td>
</tr>
</tbody>
</table>
3 DISCUSSION OF RESULTS

3.1 Beta effective, $\beta_{\text{eff}}$

We can observe in Table 1, that $\beta_{\text{eff}}$ does not depend significantly on the fuel type. This is expected as the cores with different fuel types are of practically the same size; A, B, C and D rings are completely full and E ring is only partially filled with fuel elements. Schematic view of the TRIGA core is shown in Figure 3.
In order to investigate the effect of the core size on $\beta_{\text{eff}}$ we modelled several different cores, differing in number of fuel elements. In the first core A, B and C ring were completely filled with fuel elements. In the second, third and fourth core fuel elements in D, E and F ring were added, respectively. The calculated values of $\beta_{\text{eff}}$ are presented in Table 2. It can be observed that $\beta_{\text{eff}}$ strongly depends on the core size and is bigger for smaller cores and smaller for larger cores. The result is expected as the delayed neutrons are born with lower energies and are more effective in inducing fission in systems with larger leakage.

Table 2: $\beta_{\text{eff}}$ of various uniform cores. Core description denotes the most outer ring up to which the core is filled with fuel elements.

<table>
<thead>
<tr>
<th>Core description</th>
<th>$\beta_{\text{eff}}$ (pcm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C ring</td>
<td>864 ± 14</td>
</tr>
<tr>
<td>D ring</td>
<td>839 ± 11</td>
</tr>
<tr>
<td>E ring</td>
<td>767 ± 10</td>
</tr>
<tr>
<td>F ring</td>
<td>745 ± 10</td>
</tr>
</tbody>
</table>

The effect of nuclear data library on the calculated value of $\beta_{\text{eff}}$ was investigated for the "benchmark core" [3], fuelled with 12 wt. % fuel only. The results are presented in Table 3. We can see that the calculated value of $\beta_{\text{eff}}$ strongly depends on the nuclear data library used. However, the differences do not stem from the cross-section part of the libraries as it could be expected due to the evolution of the nuclear data evaluation process in recent decades. The main differences are in delayed neutron data, as can be seen in Table 4. It is puzzling that the differences between $\beta$ values in different libraries (up to 7%) are greater than their experimental error as $\beta$ of $^{235}\text{U}$ was very accurately measured already in the fifties. It is interesting to note that the delayed neutron fraction, $\beta$, in the most recent up to date nuclear data libraries such as ENDF/B-VII and JEFF 3.1 is the same as Keepin’s [6] value from 1957, that is 0.0065.

Table 3: $\beta_{\text{eff}}$ and $k_{\text{eff}}$ of the benchmark core (Jeraj, Ravnik, 1999), that is the core fuelled with 12 wt. % fuel, calculated by using different nuclear data libraries.

<table>
<thead>
<tr>
<th>Nuclear data library</th>
<th>$k_{\text{eff}}$</th>
<th>$\beta_{\text{eff}}$ (pcm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ENDF/B-VI.8</td>
<td>0.99880 ± 0.00008</td>
<td>796 ± 12</td>
</tr>
<tr>
<td>ENDF/B-VII</td>
<td>1.00588 ± 0.00008</td>
<td>752 ± 12</td>
</tr>
<tr>
<td>JEFF 3.1</td>
<td>1.00193 ± 0.00008</td>
<td>799 ± 12</td>
</tr>
</tbody>
</table>

Table 4: Delayed neutron yields ($\nu_d$), average number of neutrons released per fission ($\nu$) and delayed neutron fraction ($\beta$) from thermal fission in $^{235}\text{U}$.

<table>
<thead>
<tr>
<th>$^{235}\text{U}$</th>
<th>Keepin (1965)</th>
<th>ENDF/B-VII</th>
<th>ENDF/B-VI.8</th>
<th>ENDF/B-V</th>
<th>ENDF/B-IV</th>
<th>JEFF-3.1</th>
<th>JEF-2.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu_d$</td>
<td>0.01580</td>
<td>0.01585</td>
<td>0.01670</td>
<td>0.01670</td>
<td>0.01670</td>
<td>0.01620</td>
<td>0.01655</td>
</tr>
<tr>
<td>$\nu$</td>
<td>2.43000</td>
<td>2.43670</td>
<td>2.43670</td>
<td>2.43670</td>
<td>2.41880</td>
<td>2.43620</td>
<td>2.43739</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.00650</td>
<td>0.00650</td>
<td>0.00685</td>
<td>0.00685</td>
<td>0.00690</td>
<td>0.00665</td>
<td>0.00679</td>
</tr>
</tbody>
</table>

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3.2 Mean neutron generation time

The mean generation time varies from 28 $\mu$s to 48 $\mu$s for 30 wt. % or 8.5 wt. % fuel, respectively. It can be observed that the mean generation time decreases with increasing $^{235}$U content. If we plot the mean generation time versus inverse $^{235}$U atom density we can observe that mean generation time is approximately linearly proportional to inverse $^{235}$U atom density.

4 CONCLUSIONS

Accurate calculations of the reactor kinetic parameters are very important for safe reactor operation. It was shown that $\Lambda$ strongly depends on the fuel type and less on the core size. The opposite holds for $\beta_{\text{eff}}$, which strongly depends on the core size and less on the fuel type. However large differences between $\beta_{\text{eff}}$ values arise also from differences in nuclear data. Therefore there is a need for improvement of calculation methods for kinetic parameters and improvement of nuclear data.

REFERENCES


[2] Chadwick, M.B. et al., 2006. ENDF/B-VII.0: Next Generation Evaluated Nuclear Data Library for Nuclear Science and Technology, Nuclear Data Sheets, Special Issue on Evaluated Nuclear Data File ENDF/B-VII.0


Fig. 1. Mean neutron generation time as a function of inverse $^{235}$U atom density.


