Boundary Conditions for the Ion Component Fluid Velocities at the Magnetic Presheath Entrance in Multi-Component Plasmas with $E \times B$ and Diamagnetic Drifts

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ABSTRACT

Boundary conditions for the ion component fluid velocities at the magnetic presheath entrance in multi-component plasmas with $E \times B$ and diamagnetic drifts are derived from the basic fluid equations. These conditions take into account gradients of the drift velocities and can be incorporated into multi-fluid codes used for modelling tokamak boundary plasmas and other magnetically confined plasmas.

1 INTRODUCTION

The next major goal of the world-wide nuclear fusion effort is a construction and operation of ITER, the next-generation experimental facility of the tokamak type. The design and plans for operation of ITER, prepared in the late 1990’s, must be continuously updated and optimised as new knowledge becomes available. Hence, developing and improving analytical and numerical tools for predictive and interpretative analyses of fusion plasmas is still of primary and even of increasing importance. Since ITER will be a single-null divertor tokamak, correct modelling of the boundary conditions at the divertor target plates is prerequisite for realistic simulations of the power and particle exhaust at the divertors.

In the multi-fluid codes used for simulating fusion edge plasmas, such as SOLPS-B2 [1], EDGE2D [2], UEDGE [3], etc., the boundary conditions at the divertor target plates are imposed at the so-called magnetic presheath entrance (MPSE) [4], where the fluid equations on which these codes are based cease to be valid. For the ion fluid velocity at the MPSE, an “intuitive” boundary condition is usually applied [4]-[6]. Assuming negligible variations of the plasma parameters in the $x$-direction in figure 1, this condition can be written as

$$v_{z,\text{MPSE}} = v_{\parallel,\text{MPSE}} \sin \alpha - \left( \frac{E_y}{B} - \frac{\partial_y P_i}{en_i B_{\text{MPSE}}} \right) \cos \alpha = c_{s,\text{MPSE}} \sin \alpha$$ (1)
Figure 1: Geometry of the model.

where \( v_{M,\text{MPSE}} \) and \( v_{z,\text{MPSE}} \) are the ion fluid velocity components parallel to \( \vec{B} \) and perpendicular to the wall, respectively, and

\[
c_s,\text{MPSE} = \sqrt{\frac{k_B}{m_i}} \left( \frac{\gamma_i T_i}{\gamma_i + T_e^*} \right)_{\text{MPSE}}
\]

is the ion-acoustic velocity at the MPSE, where \( k_B \) is Boltzmann’s constant, \( T_i \) is the ion temperature, \( m_i \) is the ion mass, \( \gamma_i \) is the ion polytropic coefficient (equalling 1 for isothermal ion flow, 5/3 for adiabatic ion flow with isotropic pressure, and 3 for one-dimensional adiabatic ion flow), and

\[
T_e^* = e n_e / \left( k_B \frac{dn_e}{d\Phi} \right)
\]

is the electron “screening” temperature, with \( n_e \) the electron density and \( \Phi \) the electric potential. The first and the second term in the parentheses are components of the \( \vec{E} \times \vec{B} \) drift velocity, \( \vec{v}_{E,B} = \vec{E} \times \vec{B} / B^2 \), and of the diamagnetic drift velocity, \( \vec{v}_{\nabla p_i} = \vec{B} \times \nabla p_i / (e n_e B^2) \), respectively, both calculated at the MPSE. \( E_y \) and \( \partial_y p_i \) are the electric field and ion pressure gradient components parallel to the wall, respectively, and \( n_i \) is the ion density. For Boltzmann-distributed electrons,

\[
n_e(\Phi) = n_{e,\text{MPSE}} \exp \left( \frac{e\Phi}{k_B T_e} \right),
\]

where \( n_{e,\text{MPSE}} \) is the electron density at the MPSE and \( T_e \) is the thermodynamic temperature, \( T_e^* = T_e \). However, besides the fact that the boundary condition (1) was not derived from the basic fluid equations, it can yield some unphysical solutions when incorporated into the fluid codes, such as highly supersonic ion fluid velocity at the MPSE or ion flow in the wrong direction, i.e., from the magnetic presheath or the wall towards the plasma presheath. Tskhakaya and Kuhn have derived the boundary condition for the ion fluid velocity at the
MPSE for two special cases, i.e., for a boundary plasma with the $\vec{E} \times \vec{B}$ drift, consisting of electrons and only one positive ion component [7] and for a multi-component boundary plasma with several positive ion components (or species) but without the drifts [8]. In this paper we generalise these studies of Tskhakaya and Kuhn and present an analysis of the MPSE in multi-component plasma with $\vec{E} \times \vec{B}$ and diamagnetic drifts, from which the boundary conditions for the ion component fluid velocities at the MPSE are derived from the basic fluid equations. These conditions take into account gradients of the drift velocities and can be incorporated into the multi-fluid codes used for modelling tokamak boundary plasmas and other similar magnetically confined plasmas.

2 ANALYSIS OF THE MAGNETIC PRESHEATH ENTRANCE

In our analysis, the planar magnetic presheath is modelled as semi-infinite plasma in front of an infinite plane surface (figure 1), with all plasma parameters depending only on the spatial coordinate perpendicular to the latter, $z$. This approximation may be used for modelling boundary plasmas if the spatial variations of the plasma parameters in the direction parallel to the wall are much smaller than the variations perpendicular to the wall. However, in contrast with Riemann’s similar magnetic presheath model [9], we take into account the electric field and ion pressure gradient components parallel to the wall, $E_y$ and $\partial_y p_i$, respectively. The magnetic field vector $\vec{B}$ lies in the x-z plane, at an angle $\alpha$ with the x-axis, i.e., $\vec{B} = B(\cos \alpha, 0, \sin \alpha)$. The continuity equation, the momentum balance equations, and the thermodynamic closure relation for each positive ion component (index $i$) can then be written as

$$\partial_z \left( n_i v_{i,z} \right) = 0, \quad (5)$$

$$v_{i,z} \partial_z v_{i,x} = \omega_{B,i} v_{i,y} \sin \alpha, \quad (6)$$

$$v_{i,z} \partial_z v_{i,y} = \frac{Z_i e E_{y,i}}{m_i} + \omega_{B,i} \left( v_{i,z} \cos \alpha - v_{i,x} \sin \alpha \right), \quad (7)$$

$$v_{i,z} \partial_z v_{i,x} = \frac{Z_i e E_{x,i}}{m_i} - \frac{\partial_z p_i}{m_i n_i} - \omega_{B,i} v_{i,y} \cos \alpha, \quad (8)$$

$$\nabla p_i = \gamma \kappa_B T_i \nabla n_i, \quad (9)$$

respectively, where $\omega_{B,i} = Z_i e B / m_i$ is the ion gyro-frequency and

$$E_{y,i} = E_y - \frac{\partial_y p_i}{Z_i e n_i}. \quad (10)$$

$E_{y,i}$ may be interpreted as the component parallel to the wall of the “effective” electric field acting on ion component $i$. (We introduced this quantity into the model in order to simplify some rather lengthy expressions.) Considering the quasi-neutrality condition,
and the definition of the electron screening temperature (3), the electric field can be expressed as

\[ E = \frac{k_B T_e^*}{en_e} \sum_i Z_i \nabla n_i. \]  

The assumption that \( n_e(\Phi) \) in the magnetic presheath equals the Boltzmann distribution (4) can be a good approximation, at least along the magnetic field lines, if the electric potential drop between the MPSE and the wall is greater than \( k_B T_e/e \). It also implies small electron flux to the wall, which cannot trigger macroscopic plasma instabilities. However, this approximation may not be so good if \( B \) is parallel or almost parallel to the wall.

The solutions of the above set of equations are formally defined for \( z \in [-\infty, 0] \), with the MPSE located at \( z \to -\infty \) and the Debye sheath entrance located at \( z \to 0 \). However, due to strong deviation from the quasi-neutrality condition (11) and strongly non-Maxwellian ion velocity distributions near the electrostatic sheath, the physical validity of these equations is limited to the region near the MPSE. Investigation of the region in the vicinity of the electrostatic sheath requires a kinetic model. We will assume the so-called “sheath-limited” transport regime near the MPSE [4], so that the electron and the ion temperatures do not vary spatially there. Stangeby noted that in the sheath-limited regime the plasma flow is approximately isothermal in the direction parallel to \( B \) in the following two cases [4]:

- when the parallel heat conductivity is very high and, hence, very weak temperature gradients can be sufficient to carry all the power which entered the boundary plasma, so regardless of whether significant plasma flow in the direction parallel to \( B \) and parallel heat convection exist or not, the plasma flow is approximately isothermal; and
- even if the parallel heat conductivity is low, if it happens that most of the particles enter the boundary plasma far upstream from the wall, the parallel convection can carry most of the power, which also reduces the parallel temperature variation, thus strong flows flatten the parallel temperature profiles.

In addition to this explanation, the assumption of isothermal flow can also be justified in tokamak boundary plasmas due to the anomalously high turbulent heat transport perpendicular to \( B \), which can approximately compensate the cooling of the ion flow when it is accelerated towards the wall. For the sake of generality of our model, we will keep \( \gamma_i \) and \( T_e^* \) (instead of \( T_e \)) in the equations, considering them as constants.

Thus, using equations (5), (9) and (12), equation (8) can be re-written as

\[
\left( \frac{v_{i,z} - c_i^2}{v_{i,z}} \right) \frac{\partial}{\partial z} v_{i,z} = -\omega_{B,i} v_{i,z} \cos \alpha - \frac{Z_i k_B T_e^*}{m_i n_e} \sum_{j=1} Z_j \frac{\partial}{\partial z} n_j ,
\]

where

\[
c_i = \sqrt{\frac{k_B}{m_i} \left( \gamma_i T_i + Z_i^2 T_e^* \frac{n_i}{n_e} \right)}.
\]
Our analysis will be based on a two-scale approximation [4], in which the quasi-neutral plasma region is divided into two distinct regions,

- the plasma presheath, or just the plasma itself, whose characteristic scale length $l_{PS}$ is the smallest of the following lengths: (i) ion collision mean-free-path, (ii) ionisation length, (iii) curvature radius or (iv) size of the plasma, and

- the magnetic presheath, whose characteristic scale length $l_{MP}$ is of the order of a few gyro-radii (or, in the multi-component plasmas, the hybrid length defined in equation (17)).

This subdivision of a quasi-neutral boundary plasma into the plasma presheath and magnetic presheath is reasonable only if $l_{MP} \ll l_{PS}$, and the electric field component perpendicular to the wall in the plasma presheath $E_{z,PS}$ is much weaker than the one in the magnetic presheath $E_{z,MP}$, i.e., $E_{z,PS} \ll E_{z,MP}$. (Otherwise, the boundary plasma should be treated as a single region, as was done in Riemann’s study [9].) Thus, we will assume that $E_{z,MP} = 0$, because in the plasma presheath the electric field component perpendicular to the wall is much weaker than the one in the magnetic presheath. Then, according to equations (11) and (12),

$$\partial_z n_{e,MP} = \sum_i Z_i \partial_z n_{i,MP} = 0, \quad (15)$$

and the boundary conditions for the ion component fluid velocities at the MPSE are given by

$$v_{i,x,MP} = v_{i,x,MP} \cos \alpha + \frac{E_{y,x,MP}}{B} \sin \alpha, \quad v_{i,y,MP} = 0, \quad (16)$$

However, it should be emphasized that the boundary conditions (15) and (16) are not correct if $\vec{B}$ is parallel or almost parallel to the wall. In that case, the charged particle fluxes to the wall are mainly controlled by the diffusion perpendicular to $\vec{B}$, which is proportional to the density gradient [4]. Therefore, our model will not be adequate if $\alpha \to 0^\circ$.

The magnetic presheath model equations and their boundary conditions can be rewritten in dimensionless form by introducing the dimensionless quantities

$$\delta = \tan \alpha, \quad \xi = \frac{z}{l}, \quad \rho_{i,x} = \frac{c_{i,MP}}{\omega_{B,i} \cos \alpha}, \quad l = \sqrt{\prod_{j=1}^{N} \rho_{i,x}},$$

$$a_i = \frac{l}{\rho_{i,x}}, \quad N_i = \frac{n_i}{n_{e,MP}}, \quad \vec{V}_i = \frac{\vec{v}_i}{c_{i,MP}}, \quad C_i = \frac{c_i}{c_{i,MP}},$$

$$U_i = \frac{a_i E_{y,i}}{B c_{i,MP} \cos \alpha}, \quad K_i = \left( \frac{Z_i T_e^*}{\gamma_i T_i + Z_i^* T_e N_i / N_e} \right)_{MP} \sum_{j=1}^{N} Z_j \partial_z n_j N_j, \quad (17)$$

where $N$ in the expression for the hybrid length $l$ is the total number of positive ion components in the plasma. Using these dimensionless quantities, the magnetic presheath equations (5)-(7) and (13) can be written as

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\[
\partial_\xi \left( NV_{i,z} \right) = 0 ,
\]
(18)

\[
V_{i,z} \partial_\xi V_{i,x} = \delta a_i V_{i,y} ,
\]
(19)

\[
V_{i,z} \partial_\xi V_{i,y} = U_i + a_i \left( V_{i,z} - \delta V_{i,x} \right) ,
\]
(20)

\[
\left( V_{i,z} - \frac{C_i^2}{V_{i,z}} \right) \partial_\xi V_{i,z} = K_i - a_i V_{i,y} .
\]
(21)

The dimensionless boundary conditions at the MPSE are given by

\[
\sum_i Z_i N_{i,\text{MPSE}} = 1 , \quad \sum_i Z_i \partial_\xi N_{i,\text{MPSE}} = 0 , \quad V_{i,x,\text{MPSE}} = V_{i,y,\text{MPSE}} \cos \alpha + \delta U_{i,\text{MPSE}} \cos^2 \alpha / a_i , \quad V_{i,z,\text{MPSE}} = V_{i,z,\text{MPSE}} \sin \alpha - U_{i,\text{MPSE}} \cos^2 \alpha / a_i .
\]
(22)

After multiplying equation (19) by \( V_{i,x} \) and multiplying equation (20) by \( V_{i,y} \), then adding these two equations and eliminating \( V_{i,y} \) from the sum by means of equation (21), we obtain

\[
\partial_\xi \left( V_{i,x}^2 + V_{i,y}^2 + V_{i,z}^2 \right) = 2 \frac{\partial_\xi V_{i,z}}{V_{i,z}} \left[ C_i^2 + \frac{U_i}{a_i} \left( \frac{C_i^2}{V_{i,z}} - V_{i,z} \right) \right] + 2K_i \left( 1 + \frac{U_i}{a_i V_{i,z}} \right) .
\]
(23)

Integration of equation (23) yields

\[
V_{i,x}^2 + V_{i,y}^2 + V_{i,z}^2 - V_{i,x,\text{MPSE}}^2 - V_{i,z,\text{MPSE}}^2 = 2 \int_{V_{i,x,\text{MPSE}}}^{V_{i,z}} \left[ C_i^2 + \frac{U_i}{a_i} \left( \frac{C_i^2}{V_{i,z}} - 1 \right) + \frac{K_i}{\partial_\xi V_{i,z}} \left( 1 + \frac{U_i}{a_i V_{i,z}} \right) \right] dV_{i,z} .
\]
(24)

We have assumed that \( V_{i,z}(\xi) \) is a monotonic (i.e., non-oscillatory), continuous, and differentiable function at least locally near the MPSE, so that the integration variable on the right-hand side of equation (24) could be \( \tilde{V}_{i,z} \) instead of \( \tilde{\xi} \). In the next step, we will express \( V_{i,x} \) as a function of \( V_{i,z} \) by means of equations (19) and (21). After multiplying equation (21) by \( \delta \), adding it to equation (19), and then integrating the sum of these two equations, we obtain

\[
V_{i,z} = V_{i,x,\text{MPSE}} + \delta \int_{V_{i,x,\text{MPSE}}}^{V_{i,z}} \left( \frac{C_i^2}{V_{i,z}^2} - 1 + \frac{K_i}{V_{i,z} \partial_\xi V_{i,z}} \right) dV_{i,z} .
\]
(25)

Upon inserting this expression for \( V_{i,x} \) into equation (24), \( V_{i,y}^2 \) can be expressed as a function of \( V_{i,z} \) , i.e.,
\[
V_{i,z}^2 = F(V_{i,z}) \geq 0 ,
\]

where \( F(V_{i,z}) \) is given by

\[
F(V_{i,z}) = V_{i,z,MPSE}^2 - V_{i,z}^2 + 2 \int_{V_{i,z,MPSE}}^{V_{i,z}} \left[ \frac{C_i^2}{V_{i,z}^2} + \frac{U_i}{a_i} \left( \frac{C_i^2}{V_{i,z}^2} - 1 \right) + \frac{K_i}{V_{i,z}} \left( 1 + \frac{U_i}{a_i V_{i,z}} \right) \right] d\tilde{V}_{i,z} - \\
- \delta^2 \left[ \int_{V_{i,z,MPSE}}^{V_{i,z}} \left( \frac{C_i^2}{V_{i,z}^2} - 1 + \frac{K_i}{V_{i,z}} \right) d\tilde{V}_{i,z} \right]^2 - 2 \delta V_{i,z,MPSE} \int_{V_{i,z,MPSE}}^{V_{i,z}} \left( \frac{C_i^2}{V_{i,z}^2} - 1 + \frac{K_i}{V_{i,z}} \right) d\tilde{V}_{i,z} .
\]

The Taylor series expansion of \( F(V_{i,z}) \) at the MPSE, i.e., about \( V_{i,z} = V_{i,z,MPSE} \), yields

\[
V_{i,z}^2 = \left( \frac{\partial^2 F}{\partial V_{i,z}^2} \right)_{MPSE} \left( V_{i,z} - V_{i,z,MPSE} \right)^2 + O\left( \left( V_{i,z} - V_{i,z,MPSE} \right)^3 \right) \geq 0 ,
\]

where \( \left( \frac{\partial^2 F}{\partial V_{i,z}^2} \right)_{MPSE} \) is given by

\[
\left( \frac{\partial^2 F}{\partial V_{i,z}^2} \right)_{MPSE} = 1 + \frac{\partial_i U_i}{a_i \partial_{\tilde{V}_{i,z}}} - \delta^2 \left[ \frac{1}{V_{i,z}^2} \left( 1 + \frac{K_i}{V_{i,z}} \right) \right]_{MPSE} \left[ \frac{1}{V_{i,z}^2} - 1 + \frac{K_i}{V_{i,z}} \right]_{MPSE} .
\]

Expressing \( \partial_{\tilde{V}_{i,z}} V_{i,z,MPSE} \) with \( \partial_{\tilde{V}_{i,z}} N_{i,MPSE} \) by means of the continuity equation (18) and replacing the dimensionless variables in equation (29) with the dimensional ones (see equation (17)), we obtain the following condition for the ion component fluid velocity at the MPSE:

\[
c_{s,i,MPSE} \sin \alpha \left( 1 + \eta_i^2 - \eta_i \right) \leq v_{i,z,MPSE} \leq c_{s,i,MPSE} ,
\]

where

\[
c_{s,i,MPSE} = \frac{k_B}{m_i} \left( \gamma_i T_i + Z_i T_e^* \frac{\partial_z \ln n_e}{\partial_z \ln n_i} \right)_{MPSE}
\]

is an “ion-acoustic-like” velocity for the ion component \( i \) at the MPSE and the drift terms are included in \( \eta_i \), defined as

\[
\eta_i = \frac{\cot \alpha \frac{\rho_{s,i,MPSE}}{L_{\eta,i}}}{2 \left( \frac{\partial_z n_e}{Z_i \partial_z n_i} + \frac{\gamma_i T_i n_e}{Z_i T_e^* n_i} \right)_{MPSE}} .
\]
where

\[ \rho_{i,j,\text{MPSE}} = \frac{c_{s,i,\text{MPSE}}}{\alpha_{B,i}}, \quad E_{y,i} = \left( \frac{\partial_z E_{y,i}}{E_{z,i}} \right)_{\text{MPSE}} = \left[ \frac{1}{E_{z,i}} \left( \frac{\partial_z E_{y,i} - \partial_z p_i}{Z_i e n_i} \right) - \frac{n_e \partial_z p_i}{Z_i^2 n_i^2 k_B T_{e,\text{MPSE}}} \right], \]

\[ E_{z,i} = E_z \left( 1 + \frac{k_B T_{e,\text{MPSE}}}{e n_e E_z} \sum_{j \neq i} Z_j \partial_j n_j \right) = -\frac{k_B T_{e,\text{MPSE}}}{e n_e} \left( \sum_{j \neq i} Z_j \partial_j n_j \right). \quad (33) \]

Using the left-hand condition in relation (30) and the quasi-neutrality condition (11), the following condition valid at the MPSE is obtained:

\[ \sum_{i=1}^{N} \frac{Z_i^2 n_{i,\text{MPSE}} \left( \sqrt{1 + \eta_i^2} - \eta_i \right)^2}{m_i \left( v_{i||,\text{MPSE}} - E_{y,i,\text{MPSE}} \cot \alpha \right)^2 - \gamma_j k_B T_{e,\text{MPSE}} \left( \sqrt{1 + \eta_i^2} - \eta_i \right)^2} \leq \frac{n_{e,\text{MPSE}}}{k_B T_{e,\text{MPSE}}}, \]

\[ v_{i||,\text{MPSE}} > \frac{E_{y,i,\text{MPSE}}}{B} \cot \alpha \pm \left( \sqrt{1 + \eta_i^2} - \eta_i \right) \sqrt{\gamma_j k_B T_{e,\text{MPSE}} \cot \alpha \over m_i}. \quad (34) \]

Relations (30) and (34) together define the MPSE in the multi-component plasma with the drifts. Additional analysis of the MPSE from the collisional presheath side (similar to the analysis in [8]) would give more precise definition of the MPSE than the above inequalities. However, for such an analysis two- or three-dimensional fluid models of the collisional presheath are necessary because the drifts are related to the spatial variations of the plasma parameters in the directions parallel to the wall. Possible kinetic effects and corrections of our multi-fluid model should also be investigated by means of two- or three-dimensional particle-in-cell simulations or kinetic codes.

3 BOUNDARY CONDITIONS FOR THE ION COMPONENT FLUID VELOCITIES

For practical implementation in the boundary conditions of the multi-fluid codes, we propose the following expression for the component-\(i\) ion fluid velocity at the MPSE, which according to relation (30) imposes the upper limit to \(v_{i,z,\text{MPSE}}\):

\[ v_{i,z,\text{MPSE}} = c_{s,i,\text{MPSE}} \left[ 1 + \left( \sin \alpha \left( \sqrt{1 + \eta_i^2} - \eta_i \right) - 1 \right) \Theta \left( 1 - \sin \alpha \left( \sqrt{1 + \eta_i^2} - \eta_i \right) \right) \right], \quad (35) \]

where \(\Theta(x)\) is the Heaviside step function. Using equations (16) and (35), the component parallel to \(\vec{B}\) and the \(x\)-component of the component-\(i\) ion fluid velocity at the MPSE can be written as

\[ v_{i||,\text{MPSE}} = c_{s,i,\text{MPSE}} \left[ \frac{1}{\sin \alpha} + \left( \sqrt{1 + \eta_i^2} - \eta_i - \frac{1}{\sin \alpha} \right) \Theta \left( 1 - \sin \alpha \left( \sqrt{1 + \eta_i^2} - \eta_i \right) \right) \right] + \frac{E_{y,i,\text{MPSE}}}{B} \cot \alpha \]

\[ \cdot \cot \alpha \]

and

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respectively, where $\alpha \neq 0^\circ$.

The multi-fluid codes mentioned in the introduction [1]-[3] calculate two-dimensional spatial profiles of the component densities, parallel velocities, ion and electron temperatures and electric potential in the poloidal plane, assuming uniformity in the toroidal direction. They can calculate all parameters included in the expressions (35)-(37), with the exception of $\gamma_i$, which is usually treated as a free input parameter. If the electric potential drop between the MPSE and the wall is greater than $k_B T_e / e$, $n_i(\Phi)$ may be approximated with the Boltzmann distribution (4), so that we may assume $T_{e,\text{MPSE}} = T_{e,\text{MPSE}}^*$. In tokamak boundary plasma modelling, values usually applied for the ion polytropic coefficient are $\gamma_{i,\text{MPSE}} = 1$ for isothermal ion flow and $\gamma_{i,\text{MPSE}} = 5/3$ for adiabatic ion flow with isotropic pressure. Isothermal ion flow is assumed in the sheath-limited regime, whereas adiabatic ion flow with isotropic pressure may be assumed if the ion velocity distributions at the MPSE are approximately Maxwellian distributions. However, such an approximation may be very poor due to a strong acceleration of the positive ions near the MPSE. Therefore, using the ion polytropic coefficient equal to the heat capacity ratio for a system in thermal equilibrium, i.e., $\gamma_i = c_{p,i} / c_{v,i}$, is wrong. Kinetic analysis of the magnetic presheath is necessary to resolve this issue.

4 CONCLUSIONS

We have derived boundary conditions for the ion component fluid velocities at the magnetic presheath entrance in multi-component plasmas with several positive ion components and both the $\mathbf{E} \times \mathbf{B}$ and diamagnetic drifts present. These boundary conditions have been derived via an analysis of the planar magnetic presheath based on the standard fluid equations, and not via an “intuitive” assumption as was the case in many of the previous pertinent studies. The main scientific motivation for this study was a need for improved, more realistic boundary conditions for the ion component fluid velocities which can be incorporated into multi-fluid codes used for modelling tokamak boundary plasmas and other similar magnetically confined plasmas.

However, there are still some open questions and problems for future studies, which have not been resolved or taken into consideration in our study. Let us mention some of them.

- Three-dimensional effects, including the electric field and ion pressure gradient components in both the $x$- and $y$-directions in figure 1, are not taken into account in our calculations. We have neglected supposedly small $x$-components of the electric field and the ion pressure gradient, which may be a wrong assumption in some boundary plasmas.
- The case of a magnetic field parallel to the wall ($\alpha = 0^\circ$ in figure 1) is not analysed in our study.
- Turbulent transport corrections of the standard fluid equations [10], i.e., additional sources of particles and momentum due to turbulent plasma fluctuations, are not taken into consideration. These effects can be very important in tokamak boundary plasmas and other similar fusion plasmas, where the turbulent transport of charged particles is mostly caused by $\mathbf{E} \times \mathbf{B}$ convection due to the fluctuating electrostatic potential [4].
Two- and three-dimensional kinetic models or particle-in-cell simulations of the magnetic presheath with self-consistent electric fields and ion pressure gradients can test the validity of our fluid model and yield some corrections that should be incorporated into the boundary conditions of the multi-fluid codes. Such kinetic models are in fact necessary to correctly investigate the regions near the Debye sheath, where the macroscopic quasi-neutrality condition (11) is significantly violated and the ion velocity distributions are strongly non-Maxwellian.

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