Sensitivity and Uncertainty Analysis for Age-Dependent Model of Test and Maintenance

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ABSTRACT

The interest in lifetime extension of existing plants is growing. Consequently, life management programs, considering safety components ageing, are being employed and developed. On the other side, the large uncertainties of the ageing parameters as well as the uncertainties associated with most of the reliability data collections are widely acknowledged.

This paper deals with sensitivity and uncertainty analysis conducted on an age-dependent unavailability model, integrating effects of test and maintenance activities, for a selected stand-by safety system in a nuclear power plant. The most important problem is the lack of data concerning the effects of ageing, which would correspond to more detailed modelling of ageing.

The obtained results indicate the extent to which the uncertainty of available ageing data sets influences the performed unavailability calculations, as well as they present sensitivity insights on the equipment.

1 INTRODUCTION

The evolution of nuclear power plant (NPP) safety depends on the evolution of the reliability of its components, which, in turn, is a function of their age along the NPP operational life [1]. Ageing of these safety components depends strongly on the surveillance and maintenance activities.

Explicit consideration of the risk effects of ageing, allowing component failure data to be evaluated for ageing effects and associated risk implications, has been an important feature of the Nuclear Plant Aging Research (NPAR) program, conducted by the Office of Research of the Nuclear Regulatory Commission [2]. Ageing of single components and simultaneous ageing of multiple components exhibited in data can be evaluated for their risk effects. Because the risk effects of ageing are not necessarily additive, the risk effects of ageing of a single component can be insignificant, but the same ageing exhibited by several components can be extremely risk significant. The risk significant ageing effects exhibited in data are of high priority and their causes need to be evaluated to assure that research programs and ageing management programs focus on these causes. On the other hand, the lack of equipment ageing data as well as the uncertainty associated with them are widely acknowledged problems. Ageing data uncertainty may implicate under- and/or overestimation of the unavailability of a system in the process of risk-informed decision-making (RIDM), e.g. risk-
informed test and maintenance (T&M) optimization [3]. Consequently, the need of incorporating ageing data uncertainties arises in the context of the mentioned RIDM process, as it is important for the decision-maker to be able to estimate how uncertain the results are and how the uncertainty associated with these ageing data is propagated in the model.

Most of the work encountered in the literature on risk-informed optimization problems does not consider ageing data uncertainty analysis. Typical uncertainties to be considered in the models are uncertainties associated with system design, which define the system reliability allocation [4], [5], [6], [7] as well as uncertainties associated with T&M activities that govern the system availability and maintainability characteristics [8], [9], [10], [11].

This paper deals with the consideration of ageing data uncertainties in an age-dependent unavailability model. A standard safety system is selected as a case study. The objective of the work is to assess the implications of ageing data uncertainties on the system unavailability calculations using the developed age-dependent model. For that purpose, a Monte Carlo (MC) simulation was used in order to follow and assess uncertainty propagation on system level. Probability density functions (PDF) of the mean system unavailability for two relevant cases were calculated and compared. Additionally, three different sensitivity analyses were run.

2 AGE-DEPENDENT T&M MODEL

The key element of the mathematical modelling of the problem is a general unavailability model as a function of STI, calculating the mean standby component unavailability including contributions of test and repair [13], [14], [15]

\[ Q_{\text{mean}} = \rho + \frac{1}{2} \lambda_0 T_i + \frac{T_r}{T_i} + (\rho + \lambda_0 T_i) \frac{T_r}{T_i} \]  

where:

- \( Q_{\text{mean}} \) mean unavailability;
- \( T_i \) test duration;
- \( T_r \) mean time to repair;
- \( \rho \) failure probability per demand;
- \( \lambda_0 \) standby failure rate;
- \( T_i \) surveillance test interval (STI).

Eq. (1) addresses the non-ageing scenario where the failure rate \( \lambda_0 \) is constant and small. It is assumed that \( \lambda_0 t < 0.1 \) where \( t \) is the time since the last test [12]. Only the linear ageing model was considered for the purpose of this paper since the ageing rates database considered in the analyses encompasses only linear ageing rates. The simpler form of the linear ageing model, where the ageing threshold is being neglected, was applied:

\[ \lambda_{\text{lin}}(t) = \lambda(t) = \lambda_0 + \theta t \]  

where \( \theta \) is linear ageing rate.

The whole development procedure of the age-dependent unavailability model used for the purpose of this paper together with the overall equations’ derivation process is presented in [3]. Component mean unavailability \( Q_{\text{mean,lin}} \), incorporating linear ageing, is given herein:

\[ Q_{\text{mean,lin}} = \rho + \frac{1}{6} T_i (3\lambda_0 + 2\theta T_i) + \frac{T_r}{T_i} + (\rho + \lambda_0 T_i) \frac{T_r}{T_i} \]  

System unavailability \( Q_{\text{sys}} \) is expressed as a function of unavailability of components. A fault tree (FT) model of the selected engineered safety system, the High Pressure Safety Injection System (HPSIS), Figure 1, of a pressurized water reactor (PWR), is used as the standpoint for the fault tree analysis [3].
Figure 1: High Pressure Safety Injection System (HPSIS)

This system is normally in stand-by and consists of three motor-driven pumps (MDPs) and seven motor-operated valves (MOVs) functionally connected with each other. Table 1 summarizes the relevant component unavailability data, as well as the time parameters associated with the T&M activities, as specified by the Technical Specifications (TS).

<table>
<thead>
<tr>
<th>Component type</th>
<th>$\lambda_0$ [h]</th>
<th>$\rho$</th>
<th>$T_s$ [h]</th>
<th>$T_{c,TS}$ [h]</th>
<th>$T_r$ [h]</th>
</tr>
</thead>
<tbody>
<tr>
<td>MDP</td>
<td>3.30E-5</td>
<td>8.40E-3</td>
<td>4</td>
<td>2190</td>
<td>72</td>
</tr>
<tr>
<td>MOV</td>
<td>5.83E-6</td>
<td>1.82E-3</td>
<td>0.75</td>
<td>2190</td>
<td>8</td>
</tr>
</tbody>
</table>

After obtaining the minimal cut sets (MCSs) as part of the FT analysis results, the MCSs are used as the standpoint for the unavailability calculation. Using a computer code developed for the purpose of the analysis, functions of the component failures are extended with addition of new time parameters concerning the contribution of test and repair - Eq. (1). Additionally, component ageing is also considered through the inclusion of time-dependent failure rates. System unavailability is estimated with the fault tree first order approximation combining $j$ MCSs, which consist of $k$ basic events (BEs) in the $j$th MCS:

$$Q_{sys}(T_s) \approx \sum_j \prod_k Q_{jk}(T_s).$$

where $Q_{jk}$ represents the unavailability associated with the BE $k$ belonging to the MCS number $j$.

3 UNCERTAINTY ANALYSIS

For the uncertainty evaluations, the linear ageing rates associated with the MDPs, $\theta_p$, and MOVs, $\theta_v$, were treated as random variables with individual log-uniform distributions [16]. The 90% error factor associated with each of the linear ageing rates, comprised in the TIRGALEX database, is taken to be a factor of 10 [16], [17]. The TIRGALEX-MOD1 database, which was constructed by modifying the purely generic TIRGALEX database to reflect results of a small study on ageing rates estimation for specific plants, was used for the uncertainty analysis. In that way, more plant specific ageing data were used instead of purely generic ones. The assignment of the log-uniform distribution was judged to be consistent with the minimal information that was available. The error factor of 10 assigned to the TIRGALEX ageing rates was retained for the pumps and valves ageing rates [16]. Thus, the ageing rate ($\theta_p = \theta_v = 1E-6$ [IFY]) multiplied by a factor of 10 gives the upper bound ($\theta_{p,UP} = \theta_{v,UP} = 1E-5$ [IFY]) and the ageing rate divided by a factor of 10 gives the lower
bound for the ageing rate ($\theta_{p,\text{LOW}} = \theta_{v,\text{LOW}} = 1E-7$ [1/hy]). In order to study and evaluate the uncertainty propagation on system level, a MC simulation-based computer code was written in MATLAB. This MC simulation consists of repeated random samplings of the linear ageing rates from their corresponding uncertainty distributions. The results presented in this section are constrained on $N=1000$ simulations per evaluation. Figure 2 presents the uncertainty propagation on system level. Figure 3 depicts the 90% confidence interval presented with the error bars. The implication of the ageing data uncertainty on $Q_{\text{sys}}$ is evident. The uncertainty impact is growing as $T_i$ rises, i.e. the obtained system unavailability is becoming more uncertain with the extension of $T_i$.

![Figure 2: Uncertainty propagation](image)

![Figure 3: 90% confidence interval](image)

Given $Q_{\text{sys}} = f(T_i)$, $T_{i,\text{opt}}$ was derived as an optimal test interval regarding the trade-off between system risk and cost in the previous work on this subject [3]. The approach and obtained results in the above referenced studies are indicating that a significant reduction in $Q_{\text{sys}}$ over a relatively small difference in total T&M costs is achievable if $T_{i,\text{TS}}$ is replaced with $T_{i,\text{opt}}$. Thus, comparative uncertainty/sensitivity analyses were done for both scenarios. Figure 4 and Figure 5 present the PDFs for the two scenarios.

![Figure 4: PDF for $Q_{\text{sys}}(T_{i,\text{TS}})$](image)

![Figure 5: PDF for $Q_{\text{sys}}(T_{i,\text{opt}})$](image)

The parameters of the PDFs above (mean $\mu$ and standard deviation $\sigma$) are given in Table 2.
Table 2: PDF parameters comparison

<table>
<thead>
<tr>
<th>PDF comparison</th>
<th>( \mu )</th>
<th>( \sigma )</th>
<th>( \sigma / \mu )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Q_{sys}(T_{i,TS}) )</td>
<td>2.059E-4</td>
<td>1.800E-5</td>
<td>8.74 %</td>
</tr>
<tr>
<td>( Q_{sys}(T_{i,opt}) )</td>
<td>2.700E-5</td>
<td>2.829E-7</td>
<td>1.05 %</td>
</tr>
</tbody>
</table>

By comparing the ratio \( \sigma / \mu \) between the two analyzed cases, \( Q_{sys}(T_{i,TS}) \) and \( Q_{sys}(T_{i,opt}) \), it is evident that dispersion from the mean value is bigger for the \( T_i \) proposed by TS, implying a higher impact of uncertainty in the respective case.

4 SENSITIVITY ANALYSIS

This section presents the results of sensitivity analysis:

- performed on the number of simulations \( N \) per evaluation of mean system unavailability \( Q_{sys} \);
- regarding the application of two additional databases as a source for the linear ageing rates in the process of deriving the \( Q_{sys} \);
- performed on selected BEs, comprised in the FT model of the HPSIS, modelling failures of MDPs and MOVs.

The dependency between the mean system unavailability \( Q_{sys} \) and the number of simulations per evaluation was studied for the two specified test intervals, \( T_{i,TS} \) and \( T_{i,opt} \) (Figure 6, Figure 7). It is evident that already for \( N=100 \) trials per evaluation in the MC simulation process, the calculated value for \( Q_{sys} \) is in the range of \( \pm 0.5\% \) of its expected value.

Figure 6: \( Q_{sys}(T_{i,TS}) \) dependency on \( N \)

Figure 7: \( Q_{sys}(T_{i,opt}) \) dependency on \( N \)

To supplement the TIRGALEX database, three additional demonstration databases were constructed, which were modifications of the TIRGALEX database. These databases were titled TIRGALEX-MOD1, TIRGALEX-MOD2 and TIRGALEX-MOD3 [16]. As mentioned in the previous section, TIRGALEX-MOD1 was constructed by modifying the initial TIRGALEX data to reflect the results of a small study that was conducted in order to estimate ageing rates for specific plants. TIRGALEX-MOD2 and TIRGALEX-MOD3 were constructed as additional sensitivity study databases to represent given types of ageing in specific components. TIRGALEX-MOD2 ageing rates represent a lower threshold ageing behaviour. TIRGALEX-MOD3 ageing database represents an upper threshold to ageing,
which is a maximum type-ageing rate that could occur for certain components. The following table, Table 3, comprises the ageing rates, associated to the considered components, for the three databases - TIRGALEX-MOD1, MOD2 and MOD3. Evaluation of the variability (in terms of relative differences) of the results of the two considered cases, \( Q_{sys}(T_{i,TS}) \) and \( Q_{sys}(T_{i,\text{opt}}) \), when the two threshold ageing databases are applied, is included in the table too.

Table 3 shows that the variability of the calculated system unavailability from the upper and lower bound, corresponding to the upper and lower ageing threshold, is substantially increasing by extension of \( T_i \).

Table 3: Sensitivity analysis considering ageing threshold data bases

<table>
<thead>
<tr>
<th></th>
<th>MOD1</th>
<th>MOD2</th>
<th>MOD3</th>
<th>( \frac{Q_{sys,MOD1} - Q_{sys,MOD2}}{Q_{sys,MOD1}} )</th>
<th>( \frac{Q_{sys,MOD1} - Q_{sys,MOD3}}{Q_{sys,MOD1}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta_p ) [( \text{hy} )]</td>
<td>1E-6</td>
<td>3E-7</td>
<td>1E-5</td>
<td>/</td>
<td>/</td>
</tr>
<tr>
<td>( \theta_s ) [( \text{hy} )]</td>
<td>1E-6</td>
<td>4E-7</td>
<td>1E-5</td>
<td>/</td>
<td>/</td>
</tr>
<tr>
<td>( Q_{sys}(T_{i,TS}) )</td>
<td>2.059E-4</td>
<td>1.931E-4</td>
<td>2.637E-4</td>
<td>6.21 %</td>
<td>28.10 %</td>
</tr>
<tr>
<td>( Q_{sys}(T_{i,\text{opt}}) )</td>
<td>2.700E-5</td>
<td>2.680E-5</td>
<td>2.788E-5</td>
<td>0.76 %</td>
<td>3.26 %</td>
</tr>
</tbody>
</table>

Sensitivity calculations for specific BEs, modelling failure of the three MDPs (“FAILURE PA”, “FAILURE PB” and “FAILURE PC”) as well as the two most risk-important MOVs (“FAILURE V24” and “FAILURE V25”), were performed as well. This analysis was performed within the PSA software [18], utilized for PSA modelling of the safety system under consideration, represented by the corresponding FT with its top event \( Q_{TOP} \) – HPSIS failure probability, i.e. system’s average unavailability. The calculations are performed in the following way:

- Set the considered BE’s probability equal to the nominal value divided by the SensFactor = 10 (default value);
- Calculate a new top event result. This new, lower, result is indicated with \( Q_{TOP,L} \);
- Set the BE’s probability equal to the nominal value multiplied by the SensFactor;
- Calculate a new top event result. This new, upper, result is indicated with \( Q_{TOP,U} \);
- Calculate the following value, which is defined as sensitivity measure: \( S = \frac{Q_{TOP,U}}{Q_{TOP,L}} \).

The results of this sensitivity analysis are presented on Figure 8. Sensitivity calculations regarding the above-mentioned BEs were carried out for three different \( T_i \)s, i.e. \( T_{i,\text{opt}} \), \( T_{i,TS} \) and one additional \( T_{i,\text{adv.}} = 4300 \) [\( \text{h} \)] \( \approx 0.5 \) years, in order for the conclusions to be more easily derived. Since the PSA software [18] does not comprise a parameter option for direct consideration of component ageing, the method of stepwise constant failure rates, [19], was utilized for incorporation of the ageing effects in the PSA model of the case study.

A conclusion can be derived that component ageing has not only a quantitative effect on mean system unavailability, \( Q_{sys} \), but also imposes a qualitative impact. Namely, while the top event sensitivity regarding one group of BEs is increasing with the extension of \( T_i \), it is decreasing in regard to other group of BEs. For smaller \( T_i \)s this sensitivity is bigger in regard to the MOVs than to the MDPs. As \( T_i \) increases, a qualitative shifting is observable, i.e. the
sensitivity in regard to the MOVs is decreasing and becomes substantially smaller than the sensitivity regarding the MDPs, which is rising.

![Figure 8: Sensitivity analysis for selected BEs](image)

5 CONCLUSIONS

The impact of the component ageing data uncertainties on system unavailability calculations, using an age-dependent unavailability model considering effects of T&M activities, is studied. Sensitivity and uncertainty analyses are carried out and the obtained results and insights are discussed.

Due to the lack of component ageing data and the large uncertainties associated with the existing ageing databases, there is a need for the decision-maker during the risk-informed decision process to be able to estimate how uncertain the results are and how the uncertainty associated with these ageing data is propagated in the model. A standard stand-by safety system, the HPSIS, was used as a case study. A MC simulation-based computer code, developed for the purpose of this paper, is used to study the component ageing data uncertainty propagation on system level. The uncertainty analysis shows that the uncertainty impact is growing with the extension of the surveillance test interval $T_i$. By comparing the probability density functions of the mean system unavailability calculated for two specific surveillance test intervals, $T_{i,TS}$ specified by the TS and the optimal one - $T_{i,opt}$ [3], it is found that the impact of the uncertainty on the results is bigger for the $Q_{sys}(T_{i,TS})$ case.

Complementary to the uncertainty analysis, sensitivity analyses are carried out as well. By studying the dependency of the mean system unavailability $Q_{sys}$ on the number of trials $N$ per evaluation in the MC simulation process, it is found that $Q_{sys}$ converges for a relatively small $N$.

Sensitivity analysis considering two ageing threshold databases is additionally run. Results show that the variability of the calculated system unavailability from the upper and lower bound, corresponding to the upper and lower ageing threshold, is substantially increasing by extension of $T_i$.

At the end, sensitivity calculations for specific BEs, modelling MDPs and MOVs unavailabilities, are performed. It is found that along with the quantitative impact the component ageing has on $Q_{sys}$, it also has a qualitative impact. For smaller $T_i$'s the top event...
sensitivity is bigger in regard to the MOVs than to the MDPs. As $T_i$ increases, a qualitative shifting is observable, i.e. the sensitivity in regard to the MOVs is decreasing and becomes substantially smaller than the sensitivity regarding the MDPs, which is rising.

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